STOCHASTIC OPTIMIZATION: AN APPLICATION TO SUB-ARCTIC DAIRY FARMING

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ABSTRACT:

The paper demonstrates how a deterministic farm linear programming (LP) model can be made stochastic and simulated using Solver and Simetar© in Excel©. The demonstration is conducted with an LP-model for a dairy farm for a sub arctic region of Norway. The income risks arising from variation in milk and crop yields due to winter damage in leys and pastures have been quantified for farms demonstrating low, medium and high forage yield risk. The estimated distribution of farm profit will be skewed to the left, indicating a downside risk. In the presence of risks, farmers maximize income by producing the milk quota with using surplus forage for meat production. The analysis demonstrated here may assist farmers and farm managers in improving sensitivity analysis for risky variables in farm LP models.

Keywords: dairy production, Northern Norway, stochastic optimization, stochastic simulation, yield risks

INTRODUCTION

Sub-arctic farming areas are defined as areas located north of the boundary at which wheat and barley may be profitable to produce (Elstrand, 1979). Dairy farming this far north, whether in Alaska, Canada or in the study area of Northern Norway, poses own risks and constraints. One constraint is the short growing season that effectively limits the number of arable crops to different grasses or some green fodder species. The pasture season lasts only a few months, so most of the milk production has to take place indoors based on locally produced roughage fodder.

Fodder yield risks due to winter damage on both meadows and pastures are caused by snow, ice carpet, or low temperatures. The effects of winter damage are a thinning of the plant carpet or even complete barren land. Farmers examine the land during springtime and determine whether some of the meadows are adequate for another year without reseeding, although the plant carpet is thin and low yields can be expected. The stochastic weather events force dairy farmers to adjust their feed ration and production levels. However, they may purchase roughage or more concentrate to keep incomes up and animal production stable. Additionally, farmers may choose to slaughter some animals prematurely to balance feed availability to production.

In response to milk yield risk the farmer generally adjusts the feeding, especially the use of concentrate. The government has kept the price of concentrate high to avoid excessive substitution of concentrate for roughage and provides extra payment for agricultural area and cultivated landscape preservation. An individual farm milk quota is another farm policy affecting dairy farms in Norway. Indoor milk production also results in a waste management problem that has to be taken into consideration in preparing a whole farm plan.

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Simulation may be an effective tool to quantify the effects of stochastic output from land and animals on farm profits or for different farms. The ration adjustment problems faced by dairy farmers further indicate that some kind of optimization is appropriate. This paper demonstrates how stochastic simulation can be combined with optimization to address economic problems of farm managers seeking to maximize profits when facing such risks.

OBJECTIVE

The general objective is to quantify variability in farm income and output due to forage and milk yield risks, taking into consideration that farmers can change their production and resource use given stochastic yields. To achieve this overall goal we address the following objectives:

1. Estimate the farm profit probability distribution for dairy farmers with a low, medium and high meadow yield, winter damage risk, and their respective milk yield risk.
2. Estimate how these risks affect their respective optimal farm production system.

The chosen methodology is a hybrid of linear programming optimization and stochastic simulation. The linear programming approach calculate the optimal farm plan given resource endowments and states of nature. Stochastic simulation offers the methodology for generating a large sample of the states of nature in a controlled experimental setting. An add-in for Excel, Simetar© allows LP models to be simultaneously simulated using stochastic constraints, input-output coefficients, and net returns per activity (Richardson, Schumann and Feldman, 2003).

A Stochastic Optimization Dairy Farm Model for the Sub-Arctic Area

Parametric LP-Model

A deterministic LP model of a family dairy farm in the three most northern counties of Norway forms the basis for this work (Asheim, Jørgensen and Havrevoll, 2001). Dairy farmers in the area also raise surplus calves for meat production. The maximization problem (Textbox 1) might be considered as a variant of maximization of gross margins as the net fixed costs are deducted in a separate process forced into the solution as an integer.

More complete model documentation with estimated income and costs is given in Asheim, Jørgensen and Havrevoll, 2000; 2001. The maximization is conducted subject to 28 constraints (Table 1).

Table 1. Number of Non-Zero Parametric Coefficients in the Deterministic LP-Model.

Constraints	Process 1-12 Roughage	Proc. 13-14 Area subsidy payment	P. 15-20 Purchase of feed	P. 21-28 Animal produce	P. 29 Hired labor	P. 30 Fixed costs	RHS
1-6 Area and manure	38			8			3
7-9 Area subsidy	12	4					2
10-13 Labor input	24			21	4	3	3
14-24 Yields/feeding	63		16	76			0
25-28 Animal output				23			7

A study (Nesheim, 1986) showed considerable amounts of old meadows in the province and a minimum meadow replacement rate was stipulated to 6%. A minimum area with winter damage is linked to processes for restoration with green fodder. The 11 feeding constraints balance the amount of energy, protein and dry matter and fibre for feeding either indoors or on pasture.

Textbox 1. The LP maximization problem of the dairy farm model.

The model maximization problem can be described as following:

$$(1)\ \text{Max } Z = \sum_{j=1}^{12} c_j X_j + \sum_{k=1}^{2} d_k Y_k - \sum_{m=1}^{6} e_m T_m + \sum_{n=1}^{8} f_n S_n - wU - F$$

where:

Z = farm[1] profit;

c_j = variable costs of meadows and pasture per hectare/10, including fuel and machinery maintenance;

X_j = area of meadows and pasture (hectare/10);

d_k = governmental support for area and landscape preservation per hectare/10;

Y_k = area qualifying for governmental landscape support;

e_m = cost of purchased feed per feeding unit[2] (FEm);

T_m = use of purchased feed;

f_n = net receipts from animals;

S_n = number of animals;

w = wage of hired labor including social costs per h;

U = hired labor, number of h;

F = net fixed costs, including interests on input of capital.

Another constraint is the milk quota. It is assumed that it is possible to produce the exact quota by several management adjustments such as early or late slaughter of old cows, adjusting the replacement, or milk feeding of calves.

The LP model was made stochastic simply by assigning the stochastic values for the random variables to their respective cells in the LP matrix. This translates into making all 63 non-zero coefficients for roughage yields in Table 1 stochastic, as well as nine of the 76 animal feeding coefficients, i.e., the feed requirements of the milking cows, and one (milk yield) of the 23 animal output coefficients. Additionally, the right hand side (RHS), coefficient for green fodder area was made stochastic, as well as the revenue from milk production or costs of crops in the objective function.

Simetar generates stochastic values, which Excel uses to update the LP model's input-output coefficients, right hand sides, and returns or costs prior to invoking Solver to optimize the model automatically (Textbox 2).

After the last iteration, statistics for key output variables (KOVs) are calculated, empirical distributions are summarized, and confidence intervals computed. Simetar includes a scenario function which enables LP-models to be run with alternative decision variables while varying the stochastic variables during a simulation. The scenario function was utilized to simulate different risk situations.

Farm data

Data to develop the deterministic model comes from the farm records for 72 dairy farms from the Account Results in Agriculture and Forestry for 1997 and 1998 (NILF 1998 and 1999) . The average milk quota was 80,685 kg of milk and the area was 21.3 hectare. To make the LP results conform to the average yield and total feed production of the farm sample, the assessed relative yields of meadows and green fodder were multiplied by a calibration factor of 0.925 (Asheim, Jørgensen and Havrevoll, 2000). With this calibration, the farm profit for the deterministic LP model was within 1.1% of the farm profit for the sample.

The sample farm was stochastically optimized for three different risk situations. From a sub-sample of 48 farms with annual yield and cost data for 10 years, 3 farms were selected with different coefficients of variation (CV) for meadow yield (Table 2). The farm selected for having the lowest CV for meadow and pasture yield (A) also had the highest average yield, whereas the farm with the highest yield CV (C) had the lowest average yield. The yield calibration factor was set at 1.66 for Farm A, 1.01 for Farm B, and 0.72 for Farm C. The three farms also have different agricultural area, milk quota, labor input and fixed costs, as indicated in Table 2.

The LP-model was stochastically optimized in a second round of analyses using the assumptions on the respective farms in Table 2. Differences in farm labor efficiency were calibrated in the deterministic mode of Simetar by tuning the sample LP model to each of the individual farm data sets: As the feed requirements in the model partly depend on milk yield the model automatically recalculated feed requirements as the milk yield was simulated.

Stochastic variables

Based on the 10 years of green fodder planting and yield history for the three sample farms, the green fodder yields and plantings were modeled as Bernoulli/multivariate empirical. A Bernoulli distribution was selected to simulate fodder yields and planting because two of the farms did not suffer winter damage sufficient to replant each year. Area planted and yields were correlated, thus necessitating the use of a multivariate empirical distribution. On Farm A, green fodder was planted only 3 of the 10 years, on Farm C it was planted 6 of the 10 years, while it was planted each year on Farm B, but with greatly different areas being planted. Hectares planted to green fodder over the 10 year history were:

Farm A [0, 0, 1.5, 0, 0, 0, 4.5, 1.4, 0, 0],

Farm B [2.5, 2.7, 6.2, 2.5, 3.9, 7.0, 2.5, 5.9, 6.8, 3.0], and

Farm C [2.0, 0, 2.5, 5.0, 3.5, 4.0, 3.4, 0, 0, 0].

The normal area planted for green fodder each year is assumed to be the minimum observed for each farm, and variations from this value are assumed to be due to winter damage. To model area and yield as Bernoulli/multivariate empirical, the probability of winter damage was simulated as a 0 or 1 using a Bernoulli distribution with parameters of 30, 100, and 60% for Farm A, B, and C, respectively. The resulting 0 or 1 value was multiplied by the multivariate empirical random values for yield and area planted to green fodder for the respective farm. The historical correlation between area and yield for years when green fodder was planted was maintained by the multivariate simulation process. The stochastic reseeded area entered the LP model as a RHS constraint i.e. as a minimum requirement.

Meadow (or pasture) yield was modeled with an empirical distribution with deviations about the mean for Farms A and B and as deviations from trend for Farm C. Farm C had a statistically significant trend, at 95% confidence level for meadow yield so the effect of the trend was removed prior to estimating parameters for the distribution. In the absence of a trend for Farms A and B, parameters were estimated using deviations from mean. Stochastic yields of meadows, pasture, and green fodder enter the model as available feedstuff for the cattle.

Milk yield per cow was also modeled with an empirical distribution. All three farms had a negative trend in milk yield; however, only Farm A had a statistically significant trend at the 95% confidence level. The stochastic milk yield for Farm A was modeled as deviations from the trend, whereas for the other farms milk yield was modeled as deviations from average yield. This variable enters the LP model in three ways: milk revenues are altered when milk yield changes, next feeding requirements change and the optimal method for meeting the milk quota to maximize net returns changes.

Model Validation

The stochastic variables in the model were simulated for 100 iterations and hypothesis tests were performed to determine if they reproduced the observed distributions. Green fodder and area planted variables were Bernoulli/multivariate empirical, so the 2 Sample Hotelling T2 test of the mean vector, the Box's M Test of the covariance matrix, and the Complete Homogeneity Test which simultaneously tests the mean vector and covariance matrix were used for validation. All three tests failed to reject the null hypothesis that the simulated mean vectors and covariance matrices equal the observed values at the 95% level for each of the three farms. Additionally, the simulated frequency of land planted to green fodder exactly equaled the observed frequency of plantings on the respective farms.

To validate meadow yields and milk production per cow, the two sample Student-t Test and the F test were used to test the mean and variance respectively for variables simulated as deviations from the mean. For variables simulated as deviations from trend, a Student-t Test and Chi-Square test were used to test the simulated sample against the trend forecasted mean and the historical standard deviation. For all six yield and milk per cow variables, the simulated means and variances/standard deviations were not statistically different from their respective historical counterparts at the 95% level.

RESULTS

The stochastic LP-model was simulated and solved for 100 iterations under two model specifications; a) using the production risk on each of the three sample farms and b) modifying for the costs, labor, land area, milk quota, and production risk on the three sample farms. Table 3 summarizes the deterministic base values obtained by solving the LP model for the two specifications. The deterministic solution values are the result with all stochastic variables set at their expected values. In specification a) the optimum solution for farm profit is only slightly different, from the $21,898 for the sample (base LP) farm to $20,649 for Farm A, $21,553 for farm B, and $21,929 for Farm C. These differences are mainly due to different milk yield on the farms.

Using specification b) the deterministic LP-solutions are more variable. The expected farm profit increases considerably to $30,449 on Farm A and $30,575 on Farm C due to higher efficiency and more resources. Additionally, both farm families also have higher farm labor input than the average farm. On Farm B profit falls considerably to $16,070 due to higher fixed costs and lower family labor input, which are not compensated by higher than average resource availability or meadow yields.

Table 4 provides the summary statistics of the simulated values for output variables across the six farm model specifications. The most notable result is that on four of the six farm models the standard deviation on farm milk production was zero and on Farm C the coefficient of variation was only 1.2% for the average farm model and 7.6% for the actual farm model specification. These results indicate the optimization routine was generally able to maximize profit by producing the milk quota regardless of the forage yields or milk per cow yields, by tactically adjusting meat production, milk cow numbers, and purchased feed. Adding milk and crop yield risk to the LP model reduced the average farm profit for all six farms relative to the deterministic solution (Tables 3 and 4). Two of the average farm/model situations saw profit decline 0.6% while Farm C saw a 2.5% decrease in profit due to risk. For the actual farm model situation, profit was 0.8 to 3.2% lower under the risk scenario.

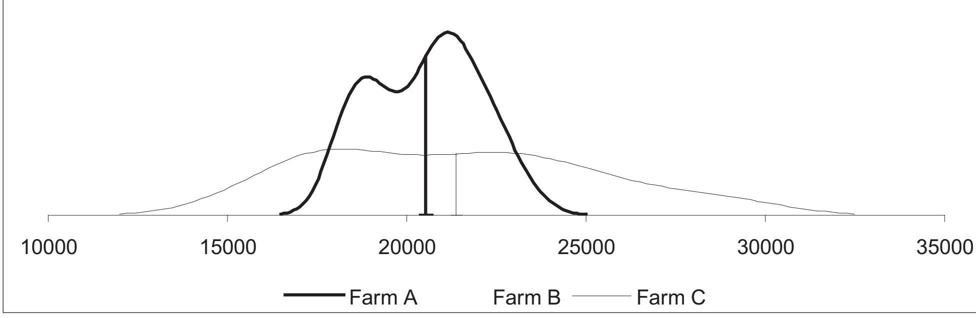
Using the actual farm's risk and resources in the basic LP model resulted in relatively more variable farm profit for two farms, B and C, than using the average farm LP model (Table 4). The CV for farm profit on Farm A is lowered from 7.2% to 4.7%, on Farm B the CV increased from 11.7% to 18.1%, and for Farm C the CV increased from 17.9% to 29.4%. These changes can be explained by resource differences between the farms outlined in Table 2.

Farm A had the highest yield and labor input, together with moderate fixed costs, and the farm area and milk quota are only slightly lower than that of the average farm. Farm B has about the same yield as the average farm, and somewhat more farm land and a larger milk quota; however, this farm has the lowest farm labor input and the highest fixed costs of the three. As a result of these differences, the average farm profit for Farm B falls when the farm's actual resource endowments are taken into consideration. On the Farm C, the mean meadow yields are the lowest of the three and the most variable. This is, however, compensated for by a considerably larger farm area, a high milk quota and high farm labor input, coupled with the lowest fixed costs of the three. The average farm profit for Farm C is similar to Farm A in spite of the low meadow yields. Increased relative risk for profit on Farm C is explained by the farm's relatively greater meadow yield risk which prevents it from producing the milk quota in all situations.

The results for the CV on meat production, milk receipts, and number of cows on Farms A and B are about the same for both methods of modelling (Table 4). For Farm C the CV for these output variables is slightly higher for the second method of modelling the farms, i.e., using actual area, fixed costs and labor inputs. Purchase of concentrate feed is the main adjustment farmers undertake when forage yields and milk per cow are risky. As can be seen from Table 4 the CV of this variable increases substantially as the forage yield risks increases from Farm A to B to C, within each model specification.

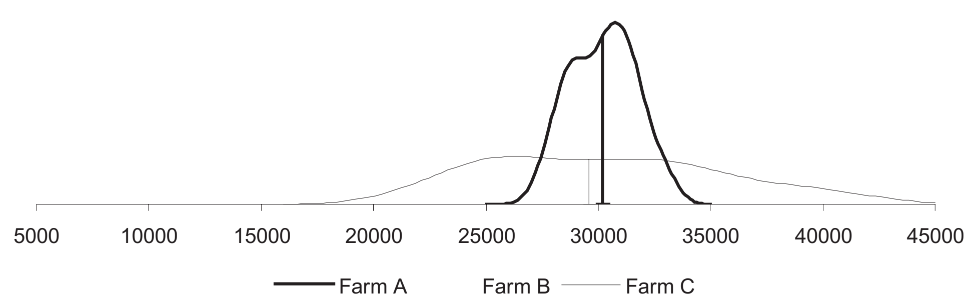
Due to differences in fixed costs among the farms, the CVs may not give a good understanding of the risks associated with farm profit of the three farms. Figure 1 and 2 portray the approximate probability density function (PDFs) of the three farms as computed for modelling specifications a) and b). The farm profit PDFs indicate a great deal of variability in income for the sample dairy farms. As the relative risk in meadow yields grows from Farm A to Farm C the

Figure 1. PDF Approximations of Net Cash Farm Income for a Sample Norwegian Dairy Farm with Risk from Three Farms ($)



PDFs spread out showing the effect of yield risk on farm profit risk. Under model specification a), the farms have similar means but different relative variability (Figure 1). However, under model specification b) the PDFs show the joint effects of different resource endowments and yield risk on each farm (Figure 2). The increased yield risk from Farm A to B is largely offset due to the resource endowments on Farm B.

Figure 2. PDF Approximations of Net Cash Farm Income for Three Norwegian Dairy Farms ($)



CONCLUSION

Firm level LP models can provide information on how inputs should be used to produce output levels that maximize profits, but do not indicate the level of income risk associated with the solution. Stochastic simulation analyzes the risk but does not allow for input or output response by the firm to the effect of risky yields, prices, or input availability. This paper demonstrates how stochastic optimization can be used to quantify income risk for a farm business that is able to tactically adjust to risk from input availability. The methodology facilitates analysis of a farm's response to risk by "annually" adjusting input use and output levels so as to maximize farm profit given risky yields and prices.

Stochastic optimization, as demonstrated here, can provide more information to producers than either deterministic LP models or stochastic firm level models. The methodology allows analysts to probabilistically project how profit maximizing farmers will likely react in terms of changing their input use and output levels in response to policy and structural changes under risk. The optimization approach alone indicates how producers should react given they know all stochastic values with certainty. Stochastic simulation methodology projects the probabilistic impacts to changes in income but without the benefit of input and output responses due to risk. The Sim/Solver function in Simetar used for the analysis can be applied to any LP or non-linear optimization model programmed in Excel. Future research efforts should be devoted to improve accounting for the different risk situations encountered.

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