MILK COMPONENTS AND FARM BUSINESS CHARACTERISTICS: ESTIMATION OF PRODUCTION FUNCTIONS VERSUS A MULTIPLE OUTPUT DISTANCE FUNCTION[1]

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Abstract:

The effects of inputs and business factors on the four milk outputs of aggregate milk, butterfat, protein, and other solids were estimated using four individual production functions, and a separate stochastic output distance function, with New York dairy farm data. Results show that 13 independent variables out of 22 display statistically significant effects on the production of at least one of the four milk components. Differential impacts of some inputs on component production indicate that milk component composition can be modified given component prices. Profit increase potentials were computed for inputs.

Keywords: farm business characteristics, milk component production, seemingly unrelated regression, multiple output distance function, technical efficiency.

Introduction

Dairy farmers in New York now receive milk payments under the Federal Milk Marketing Order multiple-component pricing system. Payments are based on the quantities of the three main milk components: butterfat, protein, and other solids. Because the price of each component is determined by the value of that milk component in processing dairy products, and ultimately the prices of final dairy products, component prices vary over time. This provides an opportunity for dairy farmers to increase profits by altering individual milk components in response to component prices.

Buccola and Iizuka (1997) estimated hedonic cost models to determine how farmers might respond to component price changes and found little opportunity to adjust components. Bailey et al. (2005) and Smith and Snyder (1978), investigated the economics of milk components by dairy breed, the factor thought most responsible for component composition. However, an important aspect that has been overlooked is the relationship between milk component production and business factors such as farm ownership type, economic scale of the farm, operator labor quality, and intensity of machinery use. Thus, the objective of this paper is to examine the effects of inputs and business factors on the four decomposed milk outputs: aggregate milk, butterfat, protein, and other solids. This is accomplished by estimating individual production functions for the four milk components, and separately a stochastic output distance function using New York dairy farm data.

Single-Output Production Functions

Estimating separate production functions for a multi-output technology requires the imposition of separability conditions. However, since all of the production factors in milk production are non-allocable, no separability assumption is imposed on outputs and inputs in estimating the four single-output

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production functions; even though farmers might want to produce only one particular component using a specific production factor, the other milk components will also be produced.

The log-linear form of the Cobb-Douglas[2] production function used in this study can be expressed as:

(1) $$\ln y_{mi} = \ln \alpha_{mo} + \sum_k \beta_{m,k} \ln x_{ki} + \sum_j \delta_{m,j} d_{ji}$$

where y_{mi} = the m^{th} output production for farm i, with m = 1 for annual milk yield per cow, m = 2 for annual butterfat yield per cow, m = 3 for annual protein yield per cow, and m = 4 for annual other solid yield per cow, x_{ki} = the k^{th} input, d_{ji} = the j^{th} categorical variable, and α_{mo}, $\beta_{m,k}$, and $\delta_{m,j}$ are coefficients.

In this model, the coefficient estimates $\beta_{m,k} = \partial \ln y_{mi} / \partial \ln x_{ki}$ represent the (partial) production elasticity of the k^{th} input for the m^{th} output. Thus, if production elasticities of the k^{th} input for the four outputs are identical, each individual component production will change proportionally according to a change in aggregate milk production, so that individual component productions, as percentages of aggregate milk, would be the same regardless of the amount of milk produced. On the other hand, if the production elasticity of each input is different for the four output productions, farmers can alter individual component productions by adjusting inputs.

Output Distance Functions

Stochastic production frontier models can be utilized in production technology with multiple outputs and inputs by incorporating a distance function (Shephard 1970; Brummer et al. 2002). If dairy farmers maximize outputs due to the difficulties of allocating inputs, the stochastic output distance function is an appropriate specification.

The log-linear form of the Cobb-Douglas output distance function for farm *i* can be expressed as

(2) $$\ln D_{oi} = \alpha_o + \sum_m \alpha_m \ln y_{mi} + \sum_k \beta_k \ln x_{ki} + \sum_j \delta_j d_{ji}$$

Because an output distance function is homogeneous of degree one in outputs, the imposition of homogeneity is accomplished by normalizing the outputs by one of the outputs. Hence, butterfat, protein, and other solids are normalized by milk production ($y^*_{mi} = y_{mi} / y_{1i}$), so that milk production becomes the dependent variable and the independent output variables are represented as percentages of each component in milk, resulting in:

(3) $$\ln D_{oi} - \ln y_{1i} = \alpha_o + \sum_m \alpha_m \ln y^*_{mi} + \sum_k \beta_k \ln x_{ki} + \sum_j \delta_j d_{ji}$$

To take into account unobserved random variations, a random error term (v_i) is added to the output distance function. $\ln D_{oi}$ is also moved to the right hand side, and replaced with u_i where u_i > 0. Then, for the purposes of easier comparison between the estimated results of the stochastic output distance function and the previous single-output production functions, the dependent variable in this equation is transformed to a positive $\ln y_{1i}$ so that the signs of the estimated coefficients will be reversed, corresponding to those in a general production function.

[2] Since 22 defined independent variables will be used, it precludes the use of more flexible functional forms that require many more estimated coefficients. However, the Cobb-Douglas is satisfactory at estimating production slopes over small data ranges, which is the case here.

(4) $\ln y_{1i} = \alpha_o + \sum_m \alpha_m \ln y^*_{mi} + \sum_k \beta_k \ln x_{ki} + \sum_j \delta_j d_{ji} + v_i - u_i$

This stochastic output distance function has two separate error terms: the symmetric random error term (v_i), and the one-sided efficiency error term (u_i). Here v_i is assumed to be independently and identically distributed, $N(0,\sigma_v^2)$, and u_i, representing the technical inefficiency, is assumed to be independently half-normally distributed, $N^+(0,\sigma_u^2)$.

The coefficient estimates $\beta_k = \partial \ln y_1 / \partial \ln x_k$ from the output distance function measures the increase in the primary output y_1, holding the output ratios of y^*_m constant. Thus, this estimated elasticity β_k includes the impact on the other outputs which keep their output ratios constant. However, by using the coefficient estimates β_k and the estimable form of the deterministic output distance function, the (partial) production elasticity of an input for each individual output can be derived. To obtain this production elasticity, first, rearrange the estimable form[3] of the deterministic output distance function as

(5) $0 = \alpha_o - (1 + \sum_m \alpha_m) \ln y_{1i} + \sum_m (\alpha_m \ln y_{mi}) + \sum_k (\beta_k \ln x_{ki}) + \sum_j (\delta_j d_{ji})$

Taking the anti-log of this function generates the equation:

(6) $1 = \alpha'_o y_{1i}^{-(1+\sum_m \alpha_m)} (\prod_m y_{mi}^{\alpha_m} \prod_k x_{ki}^{\beta_k}) e^{\sum_j \delta_j d_{ji}}$, where $\alpha'_0 = e^{\alpha_o}$

This multidimensional relationship can be represented by the general transformation function *G*.

(7) $G(x,d,y) = 0$

Then, $\partial y_1 / \partial x_k$ and $\partial y_m / \partial x_k$ can be computed by applying the implicit function theorem.

(8) $\partial y_1 / \partial x_k = -(\partial G / \partial x_k)/(\partial G / \partial y_1) = (\beta_k /(1 + \sum_m \alpha_m)) \times (y_{1i} / x_{ki})$

(9) $\partial y_m / \partial x_k = -(\partial G / \partial x_k)/(\partial G / \partial y_m) = (-\beta_k / \alpha_m) \times (y_{mi} / x_{ki})$

Multiplying Equation (8) by (x_{ki} / y_{1i}), and Equation (9) by (x_{ki} / y_{mi}) generates

(10) $\beta^o_{1,k} = \partial \ln y_1 / \partial \ln x_k = \beta_k /(1 + \sum_m \alpha_m)$

(11) $\beta^o_{m,k} = \partial \ln y_m / \partial \ln x_k = -\beta_k / \alpha_m$, where $m \neq 1$, and for $\alpha_m < 0$

These computed values represent partial production elasticities. Equation (10) measures the production elasticity of the k^{th} input x_k for aggregate milk, and Equation (11) measures the production elasticity of the k^{th} input x_k for each individual component. However, in a case where a coefficient (α_m) for an output ratio ($y^*_m = y_m / y_1$) is positive, the sign of the Equation (11) should be revised as

(12) $\beta^o_{m,k} = \partial \ln y_m / \partial \ln x_k = \beta_k / \alpha_m$, where $m \neq 1$, and for $\alpha_m > 0$.

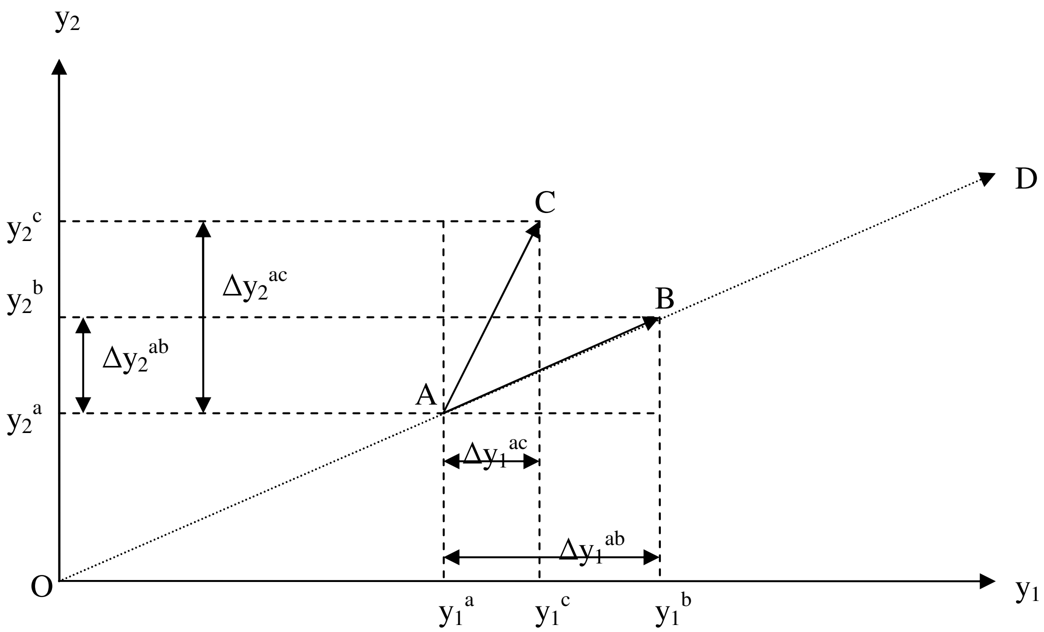
The reason is that a positive coefficient (α_m) for an output ratio ($y^*_m = y_m / y_1$) implies that milk component levels, displayed as percentages of aggregate milk, increase with aggregate milk production; this is especially true when an increase in an input causes one particular milk component to increase faster than the aggregate milk. In that case, the signs need to be the same (positive) for both the production elasticity of an input for aggregate milk (β_k) and the milk component production ($\beta^o_{m,k}$). However, Equation (11) reveals that $\beta^o_{m,k}$ and β_k have opposite signs because $\beta_k > 0$, $\alpha_m < 0$, and $\beta^o_{m,k} < 0$. Hence, Equation (12) should be used for computing the production elasticity of an input for a milk component production ($\beta^o_{m,k}$) when $\alpha_m < 0$.

[3] Equation 4 with the reversed signs of the estimated coefficients (Table 4), corresponding to those in a general production function.

The conventional way to examine the relationships between outputs is to simply look at the production possibilities curve in $y_m / y_n \ (m \neq n)$ space. Yet, unlike a business firm where the manager can alter the inputs used among various outputs, the dairy cow cannot be asked to produce different milk components from a fixed input bundle. So, in this application, a PPC in $y_m / y_n \ (m \neq n)$ space degenerates to a single point. In other words, neither the movement along the PPC in the $y_m / y_n \ (m \neq n)$ space, nor the elasticity between the outputs from the PPC in the $y_m / y_n \ (m \neq n)$ space are relevant concepts in this study. However, by increasing, decreasing, or altering that input bundle, a dairy cow might respond by producing a different output composition. Consequently, the elasticity between the outputs can be obtained by using two output combination points in multidimensional spaces rather than from the PPC. Because Δy_m, Δy_n, y_n, and y_m are easily obtained from an old output combination point and a new output combination point that is generated by a change in inputs, the elasticity between y_m and y_n $(m \neq n)$ can be calculated by $\varepsilon_{y_m, y_n} = \partial \ln y_m / \partial \ln y_n = (\Delta y_m / \Delta y_n) \ \times (y_n / y_m)$.

If the effects of input x on both outputs y_1 and y_2 are always the same, output y_2 would increase proportionally according to an increase in output y_1, so that the output ratio (y_2/y_1) is always constant regardless of the amount of outputs produced. In this case, a new output combination point will be plotted on the ray-line OD that extends out from the origin O through the old output combination point A, as shown in Figure 1. Point B is, thus, the new output combination point. On the other hand, if the effects of input x on outputs y_1 and y_2 are different, the new production point does not appear on line OD, and, in this case, point C is the new output combination point. This new output combination point C implies that by increasing the input level from x^o to x^1, farmers are able to alter individual component productions by altering inputs.

Figure 1. Two output combination points in y_1/y_2 space



To compute the elasticity between the four outputs, first rearrange Equation (5) as:

(13) $$\ln y_m = [-\alpha_o + (1 + \sum_m \alpha_m) \ln y_1 - \alpha_n \ln y_n - \alpha_l \ln y_l - \sum_k (\beta_k \ln x_k) - \sum_j (\delta_j d_j)] / \alpha_m, \text{ where } m \neq n \neq l$$

The elasticity between aggregate milk and each individual component is simply computed by taking the derivative of Equation (13) with respect to $\ln y_1$.

(14) $$\varepsilon_{y_m,y_1} = \partial \ln y_m / \partial \ln y_1 = (1+\sum_m \alpha_m)/\alpha_m$$

On the other hand, taking the derivative of Equation (13) with respect to $\ln y_n$ generates the elasticity between each individual component as Equation (15).

(15) $$\varepsilon_{y_m,y_n} = \partial \ln y_m / \partial \ln y_n = -\alpha_n / \alpha_m$$

Data

Data were obtained from the New York Dairy Farm Business Summary (DFBS) with 105 participating farms in 2003, and 107 participating farms in 2004. Although 94 of the farms submitted data for both years, estimating production functions with panel characteristics was precluded given only two years of data. Since these data were submitted on a voluntary basis, and the DFBS program is designed to assist farmers in improving their business management skills and accounting and financial analysis techniques, most of the participants are specialized commercial dairy farms. Thus, the average farm size and productivity represented in the data are larger and more productive than the average New York dairy farm. The average farm size at 448 cows is larger than the year 2004 average 95 cows New York dairy farm. Cow productivity of the DFBS farms is also higher at 21,059 lbs[4] (= 9,540 kg) of milk per cow compared to the 2004 New York state average of 17,786 (= 8,057 kg).

All outputs and inputs are measured on an accrual basis, reflecting what was actually produced and used during the year, rather than what was sold and produced. Aggregate milk and the three individual components – butterfat, protein, and other solids – are included in the model as output variables and are expressed as pounds (= 0.453 kg) per cow during a year. Outliers in the data distributions of butterfat, protein, and other solids may indicate data error, or data recorded under unusual circumstances such as diseases. Therefore, outliers were treated as missing data to prevent the possible distortion of regression results[5].

Input variables used in this study can be classified into five groups: feed, breed, labor, capital, and other managerial and environmental inputs. Among these inputs, the average herd size, bedding expense per cow per year, machinery cost per cow per year[6], BST expense per cow per year, culling rate, bred heifer rate, daily milking frequency, the operator labor quality (age, wage, and education levels), and farm ownership type are considered to be business factors in milk production. Some of these like age cannot be controlled by the farmer, others like education are mostly pre-determined, others like herd size are long-run adjustments, while many expenditures can be quickly adjusted. A year dummy variable is also included to allow for unobserved technical change and environmental aspects such as temperature and sunlight variation between the years. A summary of the variable codes used in estimation of milk component production function is provided in Table 1.

Estimation of the Four Production Functions by ISUR

The four outputs may be simultaneously affected by non-observable variables which reflect the condition of the cow. Thus, iterative seemingly unrelated regression (ISUR), which takes into account the correlations between the error terms of each equation, was used to estimate the coefficients of the four single-output production functions. The residuals from the system of equations are highly correlated: 0.82

[4] 0.453 kg

[5] Detailed criteria used to define the outliers are butterfat percentage less than 2 percent or greater than 5 percent, protein percentage less than 1 percent or greater than 5 percent, or other solids percentage less than 4 percent or greater than 8 percent. As a result of data correction, six missing data were deleted in each year.

[6] This is the sum of expenses per cow for fuel, oil, grease, machinery repairs, vehicle expense, machine hire, machine rent, machine lease, interest (5%), and depreciation.

for aggregate milk and butterfat, 0.87 for aggregate milk, and protein, and 0.92 for aggregate milk and other solids, 0.86 butterfat and protein, 0.82 for butterfat and other solids, and 0.88 for protein and other solids

Table 2 reports the different (partial) production elasticity of the k^{th} input for the m^{th} output production. This implies that the effect of this input on each output is different, so that farmers are able to alter individual component productions by adjusting this input.

Table 1: Description of Variable Names and Sample Statistics

*1 bs = 0.453 kg
**DM = Dry matter
***1 acre = 4046.8m^2

Variable names	*Description*	*Mean*	*Std. Dev.*
MILK_COW	Milk production per cow per year (in pounds*)	21306.79 (9652kg)	3366.69 (1525kg)
BUTTERFAT_COW	Butterfat production per cow per year (in pounds*)	775.25 (351kg)	105.96 (48kg)
PROTEIN_COW	Protein production per cow per year (in pounds*)	638.85 (289kg)	94.19 (43kg)
OTHERSOLIDS_COW	Othersolid production per cow per year (in pounds*)	1208.12 (547kg)	194.93 (88kg)
YEAR	Dummy for year (2003=0 and 2004=1)	105 farms in 2003 107 farms in 2004	
COWS	Average number of cows on the farm	417.68	447.39
COWS_WKR	Average number of cows per worker	38.48	12.61
OPER_AGE	Average operator age	49.08	7.61
OPER_EDU	Average operator education level	14.05	1.67
OPER_LABOR	Average operator labor contribution per cow (in months)	13.28	2.68
WAGE_MONTH	Average monthly wage for hired labor	2440.79	883.51
FORAGE_COW	Tons of home-grown forage (DM**) per cow per year	8.08	2.58
FORAGE_ACRE	Tons of home-grown forage (DM**) per acre** per year	4.07	1.24
CONCENTRATE_COW	Expense for purchased concentrate per cow per year	909.17	212.12
ROUGHAGE_COW	Expense for purchased roughage per cow per year	43.63	87.13
GENETICS_COW	Expense for genetic improvement per cow per year	46.52	26.17
COW_VALUE	Average annual cow value (in dollars)	1238.62	168.67
NON-HOLSTEIN	The percentages of Non-Holstein herds on the farm	7.98	21.76
CULL_RATE	Culling rate	32.07	7.94
HEIFER_RATE	Bred heifer rate	22.07	5.97
BEDDING_COW	Bedding expense per cow per year	49.74	39.66
MACHINERY_COW	Machinery cost per cow per year	590.56	166.86
BST_COW	BST expense per cow per year	37.87	37.23
PARLOR	Dummy for milking system type (parlor system=1)	165 farms	
SOLEOWNER	Dummy for farm ownership type (sole owner=1)	89 farms	
3xMILKING	Dummy for milking frequency (more than two times=1)	96 farms	

FORAGE_ACRE is positive in each of the four equations. This variable may be a proxy for forage quality provided to a cow. In New York higher yield per acre may represent higher quality alfalfa rather the grass. Dairy concentrate per cow (CONCENTRATE_COW) represents annual dairy concentrates, measured as an expense in dollars, and the coefficient of this variable confirms that increased expenditures for purchased concentrate per cow has major impacts on the four outputs. An increase in dairy concentrate expense leads to a relatively large increase in milk production, especially for butterfat and protein production.

The input variables related with genetics and breed, GENETICS_COW and NON-HOLSTEIN have significant effects on the four outputs. The coefficient for genetics per cow (GENETICS_COW) shows a significant impact on all four outputs. The negative coefficient for Non-Holstein breeds (NON-HOLSTEIN) shows that the Non-Holstein breed proportion on a farm results in a decrease in the quantity of aggregate milk and individual components. However, since Non-Holstein breeds produce milk containing higher butterfat and protein content as percentages of aggregate milk, the rates of decrease for butterfat and protein from a one percent increase in the percentage of Non-Holstein breeds are smaller than that of aggregate milk.

Among the variables related with the human capital of an operator such as operator age (OPER_AGE), operator education level (OPER_EDU), and operator labor contribution per cow (OPER_LABOR), only average operator age (OPER_AGE) has statistically significant effects on aggregate milk, butterfat, and other solid production, implying that the productivity of a farmer and farmer age has an inverse relationship.

Table 2: Estimation Results by ISUR

	Single-output production functions							
	***ln y_1* (aggregate milk)**		***ln y_2* (butterfat)**		***ln y_3* (protein)**		***ln y_4* (other solids)**	
	Est	***St.Err***	***Est.***	***St.Err***	***Est***	***St.Err***	***Est***	***St.Err***
D_{YEAR}	-0.0389	0.0118	-0.0574	0.0117	-0.0353	0.0114	-0.0405	0.0132
		(0.00)		(0.00)		(0.00)		(0.00)
ln (COWS)	-0.0138	0.0153	-0.0046	0.0143	-0.0147	0.0151	-0.0120	0.0163
		(0.37)		(0.75)		(0.33)		(0.46)
ln (COWS_WKR)	-0.0224	0.0322	-0.0094	0.0303	0.0203	0.0349	-0.0106	0.0352
		(0.49)		(0.76)		(0.56)		(0.76)
ln (OPER_AGE)	-0.1278	0.0359	-0.0705	0.0377	-0.0543	0.0397	-0.1114	0.0449
		(0.00)		(0.06)		(0.17)		(0.01)
ln (OPER_EDU)	0.0462	0.0468	-0.0076	0.0442	0.0526	0.0475	0.0186	0.0534
		(0.32)		(0.86)		(0.27)		(0.73)
ln (OPER_LABOR)	-0.0061	0.0284	0.0162	0.0315	0.0235	0.0279	0.0090	0.0311
		(0.83)		(0.61)		(0.40)		(0.77)
ln (WAGE_MONTH)	0.0567	0.0176	0.0386	0.0173	0.0581	0.0169	0.0542	0.0196
		(0.00)		(0.03)		(0.00)		(0.01)
ln (FORAGE_COW)	-0.0579	0.0197	-0.0668	0.0187	-0.0590	0.0193	-0.0633	0.0215
		(0.00)		(0.00)		(0.00)		(0.00)
ln (FORAGE_ACRE)	0.0879	0.0343	0.0893	0.0293	0.0752	0.0327	0.0858	0.0367
		(0.01)		(0.00)		(0.02)		(0.02)
ln (CONCENTRATE_COW)	0.1056	0.0321	0.1063	0.0324	0.1140	0.0345	0.0980	0.0326
		(0.00)		(0.00)		(0.00)		(0.00)
ln (ROUGHAGE_COW)	-0.0031	0.0038	-0.0062	0.0034	-0.0050	0.0037	-0.0017	0.0041
		(0.41)		(0.07)		(0.17)		(0.68)
ln (GENETICS_COW)	0.0552	0.0100	0.0606	0.0079	0.0574	0.0088	0.0608	0.0108
		(0.00)		(0.00)		(0.00)		(0.00)
ln (COW_VALUE)	0.0539	0.0449	0.0307	0.0405	0.0152	0.0484	0.0612	0.0491
		(0.23)		(0.45)		(0.75)		(0.21)
ln (NON-HOLSTEIN)	-0.0433	0.0056	-0.0133	0.0045	-0.0238	0.0048	-0.0395	0.0063
		(0.00)		(0.00)		(0.00)		(0.00)
ln (CULL_RATE)	0.0047	0.0294	-0.0046	0.0293	0.0094	0.0286	0.0100	0.0331
		(0.87)		(0.88)		(0.74)		(0.76)
ln (HEIF_RATE)	0.0371	0.0201	0.0158	0.0183	0.0353	0.0195	0.0379	0.0224
		(0.06)		(0.39)		(0.07)		(0.09)
ln (BEDDING_COW)	0.0102	0.0061	0.0189	0.0056	0.0126	0.0064	0.0126	0.0069
		(0.09)		(0.00)		(0.05)		(0.07)
ln (MACHINERY_COW)	0.0816	0.0282	0.0947	0.0270	0.0982	0.0261	0.0924	0.0304
		(0.00)		(0.00)		(0.00)		(0.00)
ln (BST_COW)	0.0153	0.0042	0.0129	0.0040	0.0175	0.0040	0.0149	0.0046
		(0.00)		(0.00)		(0.00)		(0.00)
D_{PARLOR}	-0.0188	0.0245	-0.0467	0.0211	-0.0486	0.0234	-0.0255	0.0270
		(0.44)		(0.03)		(0.04)		(0.34)
$D_{SOLEOWNER}$	-0.0125	0.0162	-0.0093	0.0162	-0.0116	0.0158	-0.0176	0.0173
		(0.44)		(0.57)		(0.46)		(0.31)
$D_{3\square MILKING}$	0.0941	0.0214	0.0841	0.0200	0.0958	0.0203	0.0971	0.0223
		(0.00)		(0.00)		(0.00)		(0.00)
Intercept	8.0622	0.6265	4.8246	0.5613	4.1270	0.5498	5.0272	0.6683
		(0.00)		(0.00)		(0.00)		(0.00)
	The log pseudolikelihood value of fitting constant-only model: 1217.3882							
	The log pseudolikelihood value of fitting full model: 1463.4251							

(P>z)

The negative coefficient for parlor milking system[7] (PARLOR) indicates that the parlor milking system has negative effects on butterfat and protein production. However, it is somewhat difficult to conclude that parlor milking system itself negatively affects milk production because milking system type is highly correlated with size and housing type of a farm. In the DFBS data, the correlation between parlor milking

[7] **Parlor type milking systems include the following:** Herringbone which conventional exit (46% of all parlor types), Herringbone which rapid exit (9.9%), Parallel (30.1%), Parabone (4.4%), Rotary (1.1%), and other types of parlor (8.5%).

system and freestall barn is 0.89, and those farms have an average of 526 cows. On the other hand, the correlation between stanchion milking system and tiestall barn is 0.86, and those farms have an average of 61 cows.

The average monthly wage for hired labor (WAGE_MONTH) measures the effect of hired labor quality, assuming a higher monthly wage reflects higher productivity. This was found to be true because this variable has positive effects on the four outputs. The total machinery cost per cow (MACHINERY_COW) is used as a measurement of equipment quality and non-obsolescence of that equipment, as well as the capital intensity of a farm. This variable was found to have the second most positive impact on milk production. Since bedding increases the comfort level and decreases the stress level of a cow, the bedding expense per cow (BEDDING_COW) coefficient estimates are positive for the four outputs. Superior bedding also provides a clean, dry rest area that helps prevent the spread of infectious diseases. As expected, the coefficient estimates for BST_COW and milking frequency (3×MILKING) are positive and statistically significant in the production of all four outputs.

Since some inputs are measured as expenditures in dollars and output prices are available, the effects of inputs on the milk components can be computed as the additional profit (or loss) from a one percent change in the input expenditures. This calculated additional profit represents the profit change from changes in butterfat, protein, and other solids. For instance, the total additional revenue generated using year 2005 average component prices from a one percent increase in expense for genetic expenditure per cow (GENETICS_COW) is $1.81. This is the sum of the revenues from an increase in butterfat, protein, and other solids; each revenue resulting from an increase in an individual component is equal to the component price times the amount of additional output (Δy_m), which is computed by taking the average production of each output times the production elasticity ($\beta_{m,k}$) of the input (GENETICS_COW) for that output. On the other hand, the additional cost for a one percent increase in the input (GENETICS_COW) is $0.47, which is equal to a one percent increase in the average expense for genetic expenditure per cow ($0.47 = $46.52 × 1%). Thus, $1.35 in additional profit is generated by a one percent increase in the expense for genetic expenditure per cow (GENETICS_COW). Since the profit maximization production point is where the total additional revenue equals the additional cost, farmers should spend more until the maximization condition is satisfied.

Table 3 shows that a one percent increase in GENETICS_COW, BEDDING_COW, MACHINERY_COW, and BST_COW will generate more profit for farmers; farmers can increase their profits by spending more money on bedding materials for cows, using more BST, buying more or better farm equipment related to milk production, and improving genetic traits of cows.

Table 3: Additional Profit from a One Percent Increase in Inputs

Input (x_k): GENETICS_COW, Mean x_k : $46.52								
Output (y_m)	Mean (lbs*)	$\beta_{m,k}$	Δy_m (lbs*)	Ave. Price (2005)	Add. Revenue per cow	Total Add. Revenue per cow	Add. Cost per cow	Add. Profit per cow
Butterfat	750.58 (340kg)	0.0606	0.4548 (0.206kg)	$2.46	$1.12	$1.81	$0.47	$1.35
Protein	618.33 (280kg)	0.0574	0.3549 (0.161kg)	$1.71	$0.61			
Other solids	1170.92 (530kg)	0.0608	0.7119 (0.322kg)	$0.12	$0.09			
Input (x_k): BEDDING_COW, Mean x_k : $49.74								
Output (y_m)	Mean (lbs*)	$\beta_{m,k}$	Δy_m (lbs*)	Ave. Price (2005)	Add. Revenue per cow	Total Add. Revenue per cow	Add. Cost per cow	Add. Profit per cow
Butterfat	750.58 (340kg)	0.0189	0.1419 (0.064kg)	$2.46	$0.35	$0.50	$0.49	$0.01
Protein	618.33 (280kg)	0.0126	0.0779 (0.035kg)	$1.71	$0.13			
Other solids	1170.92 (530kg)	0.0126	0.1475 (0.067kg)	$0.12	$0.02			
Input (x_k): MACHINERY_COW, Mean x_k : $590.56								
Output (y_m)	Mean (lbs*)	$\beta_{m,k}$	Δy_m (lbs*)	Ave. Price (2005)	Add. Revenue per cow	Total Add. Revenue per cow	Add. Cost per cow	Add. Profit per cow
Butterfat	750.58 (340kg)	0.0947	0.7108 (0.322kg)	$2.46	$1.75	$2.92	$0.02	$2.90
Protein	618.33 (280kg)	0.0986	0.6097 (0.276kg)	$1.71	$1.04			
Other solids	1170.92 (530kg)	0.0924	1.0819 (0.490kg)	$0.12	$0.13			
Input (x_k): BST_COW, Mean x_k : $37.87								
Output (y_m)	Mean (lbs*)	$\beta_{m,k}$	Δy_m (lbs*)	Ave. Price (2005)	Add. Revenue per cow	Total Add. Revenue per cow	Add. Cost per cow	Add. Profit per cow
Butterfat	750.58 (340kg)	0.0129	0.0968 (0.044kg)	$2.46	$0.24	$0.44	$0.38	$0.06
Protein	618.33 (280kg)	0.0175	0.1082 (0.049kg)	$1.71	$0.18			
Other solids	1170.92 (530kg)	0.0149	0.1745 (0.079kg)	$0.12	$0.02			

*0.453 kg

Stochastic Output Distance Function Results

The stochastic output distance function (4) is estimated from the maximum likelihood technique, and is reported in Table 4. The results of estimating the stochastic output distance function indicate that 12 out of 22 production factors have statistically significant effects on milk production at the 0.05 level. Since the specification of the stochastic output distance function (4) is different from the four single-output production functions, the interpretations of the coefficient estimates β_k in Table 4 are also somewhat different from $\beta_{m,k}$ [8] from the four single-output production functions. The coefficient estimates β_k from the stochastic output distance function represent the production elasticity of the k^{th} input for the overall output, holding the other inputs and percentages of each milk component y_m^* constant. Thus, this estimated elasticity β_k includes the impact on the other outputs which keep their output ratios constant. However, the resulting coefficients $\beta_{m,k}$ and β_k are almost identical; since milk components are only a

[8] The (partial) production elasticity of the k^{th} input for the particular output y_m

small portion of aggregate milk, the effects of an input on aggregate milk and on overall output are almost the same.

Table 4: Stochastic Output Distance Function Estimate

Variable	*Estimate (β_k)*	*Std. Err.*	*z*
ln (y_2^*)	-0.4904	0.1301	-3.77
ln (y_3^*)	-0.2986	0.1620	-1.84
ln (y_4^*)	0.3727	0.1672	2.23
D_{YEAR}	-0.0470	0.0116	-4.05
ln (COWS)	-0.0091	0.0139	-0.65
ln (COWS_WKR)	-0.0078	0.0291	-0.27
ln (OPER_AGE)	-0.0808	0.0380	-2.13
ln (OPER_EDU)	0.0422	0.0573	0.74
ln (OPER_LABOR)	0.0056	0.0297	0.19
ln (WAGE_MONTH)	0.0498	0.0170	2.93
ln (FORAGE_COW)	-0.0579	0.0219	-2.65
ln (FORAGE_ACRE)	0.0853	0.0292	2.92
ln (CONCENTRATE_COW)	0.1110	0.0208	5.33
ln (ROUGHAGE_COW)	-0.0051	0.0037	-1.37
ln (GENETICS_COW)	0.0553	0.0073	7.54
ln (COW_VALUE)	0.0325	0.0440	0.74
ln (NON-HOLSTEIN)	-0.0236	0.0060	-3.95
ln (CULL_RATE)	-0.0010	0.0191	-0.05
ln (HEIF_RATE)	0.0252	0.0155	1.62
ln (BEDDING_COW)	0.0143	0.0049	2.92
ln (MACHINERY_COW)	0.0934	0.0267	3.50
ln (BST_COW)	0.0143	0.0038	3.74
D_{PARLOR}	-0.0379	0.0199	-1.90
$D_{SOLEOWNER}$	-0.0064	0.0146	-0.44
$D_{3\square MILKING}$	0.0866	0.0171	5.07
Intercept	6.3761	0.8388	7.60
σ_v	0.0640	0.0168	3.80
σ_u	0.0553	0.0539	1.03
$\sigma^2_s = \sigma^2_v + \sigma^2_u$	0.0072	0.0039	1.83
$\lambda = \sigma_u/\sigma_v$	0.8646	0.0704	12.28

There appears to be very little technical inefficiency among the New York dairy farms that participated in this DFBS project. The minimum value of estimated technical efficiency is 90% and the average is 96%. These data participants represent high performance farms, so production variation between farms may be relatively small, leading to a high minimum and average efficiency. In addition, the distance function included many business factors such as those representing the economic scale of the farm and the operator labor quality, which possibly affect the technical efficiency level of a farm. In this way, the effects of technical inefficiency are captured in the coefficient estimates, instead of in a one-sided error term.

The (partial) production elasticity $\beta^{o}_{m,k}$ of each input for each of the four outputs is reported in Table 5. These elasticities are computed by using Equations (10)-(12) and the coefficient estimates for the inputs that have significant effects on milk production. The interpretations of $\beta^{o}_{m,k}$ are similar to the coefficient estimates $\beta_{m,k}$ from the four single-output production functions. The absolute values of $\beta^{o}_{m,k}$ and $\beta_{m,k}$ are slightly different, but the signs and relative effects of the production factors that significantly affect the four output production are almost identical. Thus, the impact of the factors estimated from the distance function is not significantly different from the impacts estimated from the four separate production functions discussed previously.

Table 5: Computed Production Elasticities of Inputs for the Four Outputs

	ln y_1 ***(aggregate milk)***	***ln*** y_2 ***(butterfat)***	***ln*** y_3 ***(protein)***	***ln*** y_4 ***(other solids)***
	$\beta^{o}_{1,k}$	$\beta^{o}_{2,k}$	$\beta^{o}_{3,k}$	$\beta^{o}_{4,k}$
D_{YEAR}	-0.0806	-0.0959	-0.1575	-0.1262
ln (COWS)	-0.0156	-0.0186	-0.0305	-0.0245
ln (COWS_WKR)	-0.0133	-0.0159	-0.0261	-0.0209
ln (OPER_AGE)	-0.1384	-0.1647	-0.2706	-0.2168
ln (OPER_EDU)	0.0722	0.0860	0.1412	0.1131
ln (OPER_LABOR)	0.0096	0.0114	0.0187	0.0150
ln (WAGE_MONTH)	0.0853	0.1016	0.1668	0.1336
ln (FORAGE_COW)	-0.0993	-0.1181	-0.1940	-0.1555
ln (FORAGE_ACRE)	0.1462	0.1740	0.2858	0.2290
ln (CONCENTRATE_COW)	0.1901	0.2263	0.3716	0.2977
ln (ROUGHAGE_COW)	-0.0087	-0.0103	-0.0169	-0.0136
ln (GENETICS_COW)	0.0947	0.1127	0.1851	0.1483
ln (COW_VALUE)	0.0557	0.0663	0.1089	0.0873
ln (NON-HOLSTEIN)	-0.0404	-0.0481	-0.0790	-0.0633
ln (CULL_RATE)	-0.0017	-0.0020	-0.0033	-0.0026
ln (HEIF_RATE)	0.0432	0.0514	0.0844	0.0676
ln (BEDDING_COW)	0.0245	0.0292	0.0480	0.0384
ln (MACHINERY_COW)	0.1600	0.1904	0.3127	0.2505
ln (BST_COW)	0.0246	0.0292	0.0480	0.0385
D_{PARLOR}	-0.0650	-0.0773	-0.1270	-0.1018
$D_{SOLEOWNER}$	-0.0110	-0.0130	-0.0214	-0.0172
$D_{3□MILKING}$	0.1484	0.1766	0.2900	0.2323

Finally, elasticities between the outputs computed by using Equations (14) and (15) are presented in Table 6.

Table 6: Elasticities between Outputs

	y_2 (butterfat)	y_3 (protein)	y_4 (other solids)
y_1 (aggregate milk)	-0.8402	-0.5116	0.6385
y_2 (butterfat)		-0.6089	0.7600
y_3 (protein)			1.2482

Conclusions

This study measured the responses of aggregate milk and individual milk component production to changes made in the dairy business. Four single-output production functions and a stochastic output distance function were estimated using New York Dairy Farm Business Summary (DFBS) data from 105 farms in 2003 and 107 farms in 2004. The empirical results demonstrate the possibility of altering individual component productions. However, since the differences between the effects of each input on each output are relatively small, the farmer's ability to alter individual component productions may be limited. Yet, this is still important because, given the small profit margins that often occur in the dairy industry, this small ability provides the opportunity for farmers to increase profits by altering individual component production levels in response to each component price.

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