

1

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### **Three phase feed-mix optimization for growing pigs**

#### **Abstract**

The ability to be adaptable in the process of daily ration formulation demands handy tools to support their calculation. This paper presents an example of such a tool, which is based on a three phase optimization approach. In the first phase, a common linear program is utilized to formulate rations on a least-cost basis. In the second stage, a sub-model, which is based on weighted goal programming and supported by a system of penalty functions, is used to formulate a nutritionally balanced and economically acceptable ration. This ration relies upon the ration cost calculated in the first phase. In the last stage, the tool runs the first and the second phases several times with the intent of finding the most “efficient” energy content of the ration. The obtained results confirm the benefits of the applied approach. This model enables decision makers to find the optimal energy content of the pigs’ diets, which changes frequently due to rapidly fluctuated economic circumstances.

#### **Introduction**

Pork production is globally characterized by low profits, demanding adaptable management, and adjustable production. Because the farmer’s main objective is to maximize profits, costs must be minimized. This may be accomplished through improved technical or economical efficiency. Because of the high expense of ration costs and the possibility of negative externalities that might occur, it is obvious that ration formulation is a crucial task in daily pig breeding management.

Pig production is also one of those agricultural activities that are not necessarily connected with arable land. This sector is therefore closely related to common industrial production. In particular, the majority of inputs may be exogenous, or not produced on the farm. In this case, changes in world (cereal) markets could rapidly affect the economic outcome. However, even if the majority of the feed is produced at the farm gate, there are opportunity costs that require the decision maker to make efficient decisions in relation to breeding practices. This may allow for improved productivity, or at least may keep profitability at an acceptable level. Because of rapid changes in exogenous factors and simultaneous technological improvements, this task is becoming more and more challenging. In order to help breeders to deal with these challenges, many tools based on constraint optimization have been developed.

2

Pioneering work in this area has been conducted by Waugh (1951), who applied the linear programming (LP) paradigm in order to formulate rations on a least-cost basis. This approach has been very popular in the past, especially after the rapid development of personal computers. In the 1960s, it became a classical approach to formulate animal diets as well as feed-mixes (Black and Hlubik, 1980). More recently, Castrodeza et al. (2005) stressed that the daily routine of ration formulation is one of the fields in which LP is most widely used.

Common to all LP problems is the concept of constraint optimization, which means that one tries to find the optimum of a single objective function. However, exclusive reliance just on one objective (cost function) as the only and the most important decision criteria is one of the reasons why the LP paradigm may be a deficient method in the process of ration formulation (Rehman and Romero, 1984; 1987). Lara and Romero (1994) stress that in practice, decision makers never formulate rations exclusively on the basis of a single objective, but rather on the basis of several different objectives, where economic issues are only one of many concerns.

In common LP models for pig ration formulation, animal amino acid requirements are usually expressed in terms of minimal concentrations. Such models do not consider the total exceeded amount of protein or its quality as long as the minimal amounts of essential amino acids are satisfied (Bailleul et al., 2001). The same authors stress that “economical optimal” diets are often too rich in protein, which directly burdens the environment and does not improve animal growth. This problem could partly be solved by adding additional upper or lower constraints. However, it might rapidly lead into over-constraint model that has no feasible solution. This problem is also related to the next LP drawback. It is mathematical rigidity of constraints (right hand side – RHS), which might also result in facts with no solution (Rehman and Romero, 1984). This means that no constraint (e.g. given nutrition requirements) violation is allowed at all. However, relatively small deviations in RHS would not seriously affect animal welfare, but would result in a feasible solution (Lara and Romero, 1994).

Numerous methodological developments in the field of mathematical programming (MP) have eased these problems of LP paradigm (Buysse et al., 2007). For instance in the field of animal nutrition, Rehman and Romero (1984) introduced goal programming and its improvement with a system of penalty function, as well as multi-objective programming (MOP) as a way to incorporate more than one objective function; Lara and Romero (1994) applied interactive

3

methodologies where the optimal ration is achieved through “computer dialog”; Castrodeza et al. (2005) addressed a multicriteria fractional model.

The purpose of this paper is to present a tool for pig ration formulation, designed as a three-phase optimization approach that merges two normative mathematical programming techniques. The first part of the paper provides a brief overview of weighted goal programming (WGP) and the penalty function. This is followed by a short description of the optimization tool that also involves LP in order to calculate least-cost ration formulation. Finally, the characteristics of the analysed case are presented, followed by the results and discussion.

## **Material and methods**

### **Weighted goal programming supported by a system of penalty functions**

Based on the approaches reported in the literature and the primary aim of this tool, we decided to apply the WGP approach. This was in the context of ration formulation introduced by Rehman and Romero (1984). The WGP approach is a pragmatic and flexible methodology for resolving multiple criteria decision making problems. In this case, a farmer is interested in an economically efficient ration that achieves a balance between several conflicting objectives that include cost, nutrition imbalances, and possible negative impacts on the environment (Weintraub et al., 2001).

WGP formulation is expressed as a mathematical model with a single objective (achievement) function (the weighted sum of the deviations variables). The optimal compromise solution is found through the philosophy of “distance measure” that measures the deviation between the desired goal and the performance level of a goal.

In most cases, the obtained solution is a compromise between contradictory goals, enabled with positive and negative deviation variables. Negative deviation variables are included in the objective function for goals that are of the “more is better” type, and vice versa. Relative importance of each deviation variable is determined by belonging weights.

Since the goals are measured in different units and have different numerical values, the deviations are scaled with normalization techniques (Tamiz et al., 1998). Through this process, incommensurability is prevented and all deviations are expressed as ratio differences.

Rehman and Romero (1987) point out that the main drawback of WGP concerns marginal changes. Namely, the method does not distinguish between marginal changes within one

4

observed goal; all changes (deviations) are of equal importance. This addresses an additional issue that relates to the example of ration formulation. Specifically, in some situations, too large of a deviation might result in a failure to keep the animal's diet within desirable nutritional limits, rendering the solution useless. To keep deviations within desired limits and to distinguish between different levels of deviations, a system of penalty functions (PF) needs to be introduced into the WGP model (Rehman and Romero, 1984).

This system is coupled with the achievement function (WGP) through penalty coefficients and with additional constraints defining deviation intervals. This approach enables one to define allowable positive and negative deviation intervals separately for each goal. Depending on the goal's characteristics (nature and importance of 100 % matching), these intervals might be different. Sensitivity is dependent on the number and size of defined intervals and the penalty scale utilized ( $s_i$ ; for  $i=1$  to  $n$ ).

### **Tool for three phase pig ration formulation**

This optimization tool for pig ration formulation was developed in MS Excel as an add-in application. This tool is capable of formulating least-cost, nutritionally balanced, and environmental acceptable rations for growing pigs in different production periods. It also gives information about which feed-mix provides the optimal energy content.

The tool is organized as a three phase approach that merges two sub-models based on mathematical programming techniques (Figure 1). The first sub-model is an example of a common least-cost ration formulation, based on the LP paradigm. The purpose of including this into the tool is to get an approximate estimate of expected ration cost. In this manner, the tool calculates the target economic goal, which is one of the goals in the second sub-model. The first sub-model is therefore, from the perspective of constraints, as simple as possible and is intended to exclusively measure the crude cost estimation. Through cost function, this is linked to the second sub-model. The latter is based on WGP and is supported by a system of six sided PF. In this approach, the desired nutrition levels and ration costs are modelled as goals instead of as constraints. In the second sub-model, additional constraints with indirect influence on the environment are added. Consequentially, the model is much more complex, and it finally yields a better solution-formulated ration.

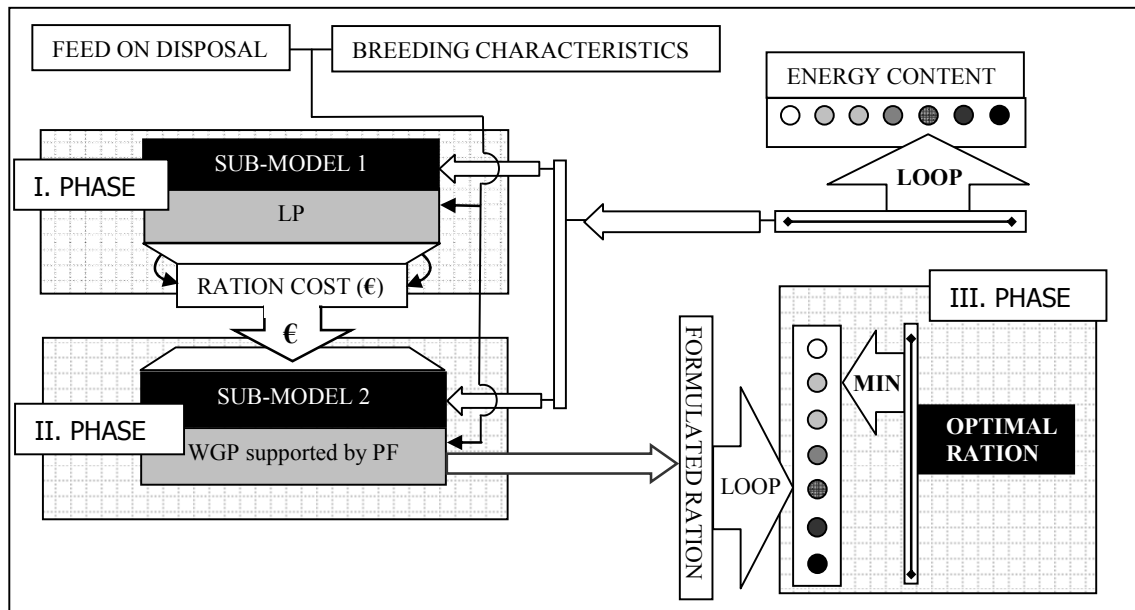


Figure 1: The scheme of the tool for three phase pig ration formulation

Because of the importance of the energy concentration of the feed-mix and its influence on the ration structure and cost, the tool also includes a third phase. In this phase, a macro loop is added that runs the first and the second sub-models for n-times, and consequentially it yields n-formulated rations. The number of iterations in the third phase depends on the starting/ending energy content of the feed-mix and on the energy rise in each iteration step (e.g. 0,1 MJ/kg). From the n-obtained solutions, the tool selects the cheapest option and marks it as the “optimal” feed-mix structure for this given example.

**Mathematical formulation of the first and the second sub-model**

The first sub-model (LP) is formulated as shown in equations (1\*), (4a), and (7). It mostly relies on economic (cost) function (C) and satisfies only the most important nutrition requirements coefficients (b<sub>i</sub>), known also as RHS. In the first optimization phase, one is searching for the ration at the lowest possible cost (C).

$$\min C = \sum_{j=1}^n c_j * X_j \quad \text{such that} \tag{1*}$$

$$\min Z = s_1 \sum_{i=1}^k w_i \frac{d_{i1}^- + d_{i1}^+}{g_i} + s_2 \sum_{i=1}^k w_i \frac{d_{i2}^- + d_{i2}^+}{g_i} \quad \text{such that} \tag{2}$$

$$\sum_{j=1}^n a_{ij} X_j + d_{i1}^- + d_{i2}^- - d_{i1}^+ - d_{i2}^+ = g_i \quad \text{for all } i = 1 \text{ to } r \text{ and } g_i \neq 0 \tag{3}$$

6

$$\sum_{j=1}^n a_{ij} X_j \leq b_i \quad \text{for all } i=1 \text{ to } m \quad (4a)$$

$$\sum_{j=1}^n (a_{ij} - (R^{\%})a_{kj}) X_j \leq 0 \quad \text{for all } i=1 \text{ to } m \quad \text{and } i \neq k \quad (4b)$$

$$d_{i1}^- \leq g_i - p_{i1}^{\min} g_i \quad \text{for all } i=1 \text{ to } r \quad (5a)$$

$$d_{i1}^- + d_{i2}^- \leq g_i - p_{i2}^{\min} g_i \quad \text{for all } i=1 \text{ to } r \quad (5b)$$

$$d_{i1}^+ \leq p_{i1}^{\max} g_i - g_i \quad \text{for all } i=1 \text{ to } r \quad (6a)$$

$$d_{i1}^+ + d_{i2}^+ \leq p_{i2}^{\max} g_i - g_i \quad \text{for all } i=1 \text{ to } r \quad (6b)$$

$$d_{i1}^+, d_{i1}^-, d_{i2}^+, d_{i2}^-, X_j \geq 0 \quad (7)$$

The second sub-model (WGP with a PF) is formulated as shown in equations (2) to (7). The achievement function ( $Z$ ), expressed in equation (2), is defined as the weighted sum of the undesired deviation variables ( $d_{i1}^+$ ,  $d_{i1}^-$ ,  $d_{i2}^+$ ,  $d_{i2}^-$ ) from observed goals ( $g_i$ ), multiplied by belonging penalty coefficients ( $s_1$  and  $s_2$ ) that measure the slope of the penalty function. The obtained sum-product is the subject of minimization (2). The relative importance of each goal is represented by weights ( $w_i$ ) associated with the corresponding positive or negative deviations. Penalty intervals ( $p_{i1}^{\min}$ ,  $p_{i1}^{\max}$ ,  $p_{i2}^{\min}$ ,  $p_{i2}^{\max}$ ) are in place to prevent uncontrolled deviations (5a to 6b) within each goal. Because of the normalization process, only goals that have nonzero target values (3) could be relaxed with positive and negative deviations. The obtained target value ( $C$ ) in the first sub-model enters into the second one (WGP with PF) through “cost goal” ( $g_i = C$ ). This is also the only case where negative deviation is not penalized and also not restricted to intervals. All other constraints that do not have defined target values or do not have priority attributes are considered in the equation (4a). All upper bounds for ratios ( $R^{\%}$ ) are transformed into linear equations with equation (4b), and the same holds for lower bounds, which should be multiplied by -1. Non-negativity condition for both models is considered in equation (7).

### Analyzed example

To illustrate the use of the tool, we chose a hypothetical case of pig production. We considered that the tool should formulate the complete ration/feed-mix in relation to the nutritional requirements. Due to significant changes in nutritional requirements throughout a pig's life, the fattening period (40 kg – 100 kg) has been split into three periods with a 20 kg weight gain in each. In each period, the average daily weight gains were different. In the first period, the pigs gained 740 g/day; in the second period, they gained 800 g/day; in the last period, they gained 750 g/day.

7

Table 1: Assumed daily requirements for three fattening periods

		Fattening period		
		40-60 kg	60-80 kg	80-100 kg
Average daily weight gain (g/day)		740	800	750
Metabolisable energy (ME)	MJ	25.5	31.4	34.8
Crude protein (CP)	g/day	330	370	340
Lysine (Lys)	g/day	17.2	19.5	18.8
Methionine + Cystine (Met + Cys)	g/day	10.3	11.7	11.3
Methionine (Met)	g/day	5.2	5.9	5.6
Threonine (Thr)	g/day	10.3	11.7	11.3
Tryptophan (Trp)	g/day	3.4	3.9	3.8

All nutrition requirements are taken from DLG (1991). The most important are presented in Table 1. Besides those, the tool also considers the mineral requirements (Ca, P, and Na). The formulated ration should also have the appropriate crude fibre content, which is assured through minimal and maximal ratio as well as protein digestibility. In order to prevent a solution that has too much of one feed in the diet, we considered recommendations for maximal feed inclusion (Futtermittelspezifische ..., 2006).

Table 2: Importance of goals with corresponding penalty function intervals

Goal	Unit (/day)	Weight ( $w_i$ )	Penalty function intervals				Together	
			$p_{i1}^+$	$p_{i1}^-$	$p_{i2}^+$	$p_{i2}^-$	$p_i^+$	$p_i^-$
			%				%	
ME	MJ	80	1	0	2	0	3	0
CP	g	65	1	0	2	0	3	0
Lys	g	75	2	1	3	3	5	4
Met + Cys, Met, Thr and Trp	g	65	3	2	5	4	8	6
$P_{digest}$	g	20	5	5	10	10	15	15
Cost	Cent	20/90	5	$\infty$	20	$\infty$	25	$\infty$

Weights indicate the decision maker's preferences with respect to each goal. The tool offers the option to switch between goals and constraints, depending on the needs of the decision maker. In the analyzed case, we chose nine goals (Table 2) that should be met as accurately as possible. The importance of each goal is defined by weights ( $w_i$ ) ranging between 0 and 100. For each goal, deviation intervals are defined separately. They are measured in percentage deviation from the desired level. The most rigorous and short intervals are anticipated by energy, protein, and amino

8

acids goals. Specifically, reducing the unbalanced protein fraction by increasing protein quality (fulfilling the amino acids ratios in relation to the energy) reduces nitrogen excretion and pollution.

The tool also includes “the library” of feeds and their nutrition values. It is organized in such a way that it is possible to change the quality parameters, as well as add new ones. The initial tool version includes 35 feeds and vitamin-mineral mixtures. In the process of ration formulation, one can select only those that are at one’s disposal to enter into the formulated ration.

### **Results**

The application of the tool is presented through a simple example that might be applicable to larger agricultural holdings. This means that rations primarily consist of feed-mixes prepared at the farm gate. We have presumed that the decision maker prepares three different feed-mixes for growing pigs, in two different scenarios. In the first scenario, the most important element is quality of the ration ( $W_{cost}=20$ ), while in the second scenario, cost is more important ( $W_{cost}=90$ ).

The results obtained are presented in Figures 2 and 3. Figure 2 illustrates the structure of two different ration sets differentiated by their cost importance. Within each set, formulated rations are dependent on the energy concentration of animal rations. The range of the energy content of the ration was set between 12.3 and 13.7 MJ ME. Figure 3 illustrates the level of ration costs for different fattening periods over the same range of ME content.



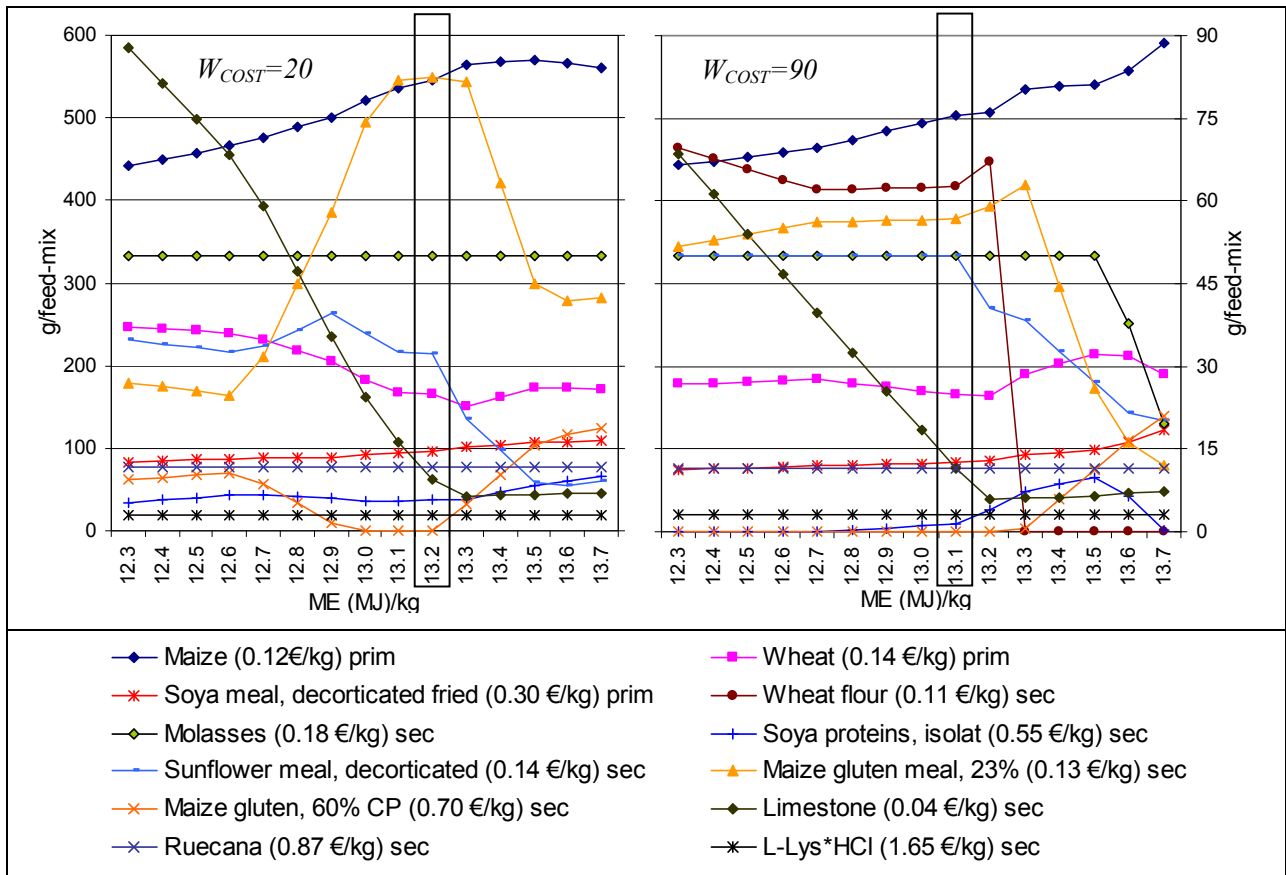


Figure 2: Formulated feed-mix for the second fattening period at low ( $W=20$ ) and high importance ( $W=90$ ) of ration cost (prim = primary axis; sec = secondary axis)

One of the main parameters that defines how much a pig is going to eat is the energy content of the feed-mix. If the feed-mix is more concentrated, an animal is going to eat less, and vice versa. Figure 2 presents formulated rations for the second fattening period. It is obvious that the energy content of the ration strongly influences selection of the feed. With increasing energy content, the quantity of maize increases and the quantity of wheat decreases. From Figure 2, it is apparent that cheap wheat flour reduces costs, since it enters into the solution only at high  $W$  values. The same holds for sunflower meal. From Figure 2, one can also observe the phenomena of energy-low rations, demonstrated by high limestone content.

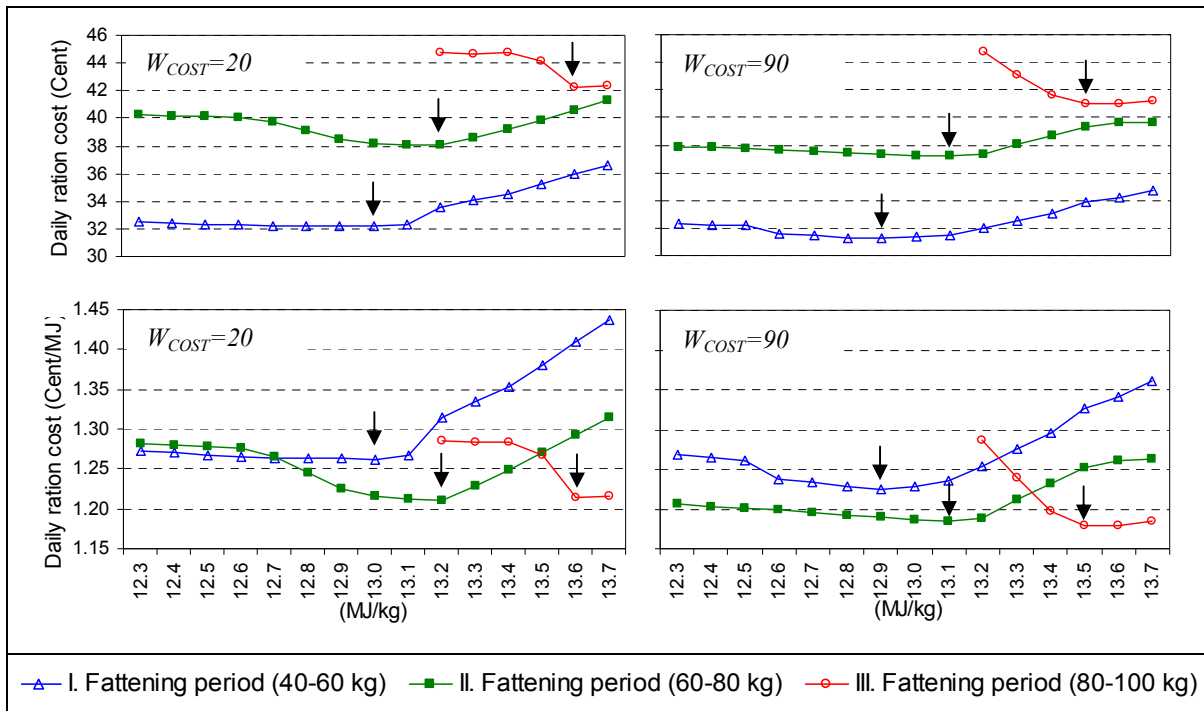


Figure 3: Daily ration costs dependent on energy content of the feed-mix by low (left side) and high (right side) ration cost importance

The importance of finding the “optimal” energy content of the feed-mix, which further influences its structure and the profitability of pork production, is illustrated in Figure 3. It is logical that pressure on economics reduces rations’ cost. The difference ranges between a few tenths of a cent up to several cents per day, and increases with animal growth. Of course, the difference for one pig per day is no big deal, but if one considers a facility with 50,000 animals, this becomes an important issue. It is an interesting coincidence that optimal rations in all three fattening periods shift to the left if the cost is of higher importance. Even though the difference is only 0.1 MJ, this demonstrates that it is cheaper to formulate feed-mixes with lower energy content. From Figure 3 it is also apparent that in spite of lower daily ration cost in earlier fattening periods, such rations are more expensive per MJ of ME.

With animal growth, the optimal energy content shifts to higher concentrations. From Figure 3, we can see that in the third fattening period, the tool finds solution only from 13.2 MJ onwards. Deviations from set goals are too big and the tool does not find solutions at lower energy.

The difference in ration costs between different energy concentrations is obvious. It ranges 2.4 up to 4.5 cents per daily ration. In any case, it should be an important issue to find the “optimal” energy concentration in the daily management of pork production.

### **Conclusions**

The results of this study show that the three phase optimization approach, supported by mathematical programming (LP and WGP with PF), can be applied efficiently to the ration formulation for growing pigs. Through using this tool, more efficient diets might be formulated, since the model enables the decision maker to find the optimal energy content of the diet for various economic circumstances. The tool could be utilized in medium and large-scale agricultural holdings, which are usually the major pollutants of the environment, and where such tasks are an important part of daily management. Specifically, precisely balanced rations prevents over-feeding as well as under-feeding, both of which are expensive and burden the environment.

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