NITROGEN FERTILIZER RECOMMENDATIONS BASED ON PRECISION SENSING AND BAYESIAN UPDATING

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Abstract

Methods are available to predict nitrogen needs of winter wheat based on plant sensing, but adoption rates by producers are low. Low adoption rates are likely due to economic feasibility. The objective of this paper was to develop a method to Bayesian update prior information about nitrogen application with precision sensing information and determine if Bayesian updated nitrogen recommendations increase the economic feasibility of precision sensing technology. Results indicated that Bayesian updated nitrogen recommendations were higher than precision sensing recommendations. However, the Bayesian nitrogen recommendations did not increase net returns.

Keywords: Bayesian updating, nitrogen response, precision sensing, winter wheat

1. Introduction

Although extensive research on production functions has decreased uncertainty for producers when making a choice, precision sensing (PS) technology attempts to further decrease uncertainty by providing contemporaneous information. However, despite large expenditures directed to research and development of PS, adoption rates by producers are still low. This is most notable for PS technology to select fertilizer application. Whipker and Akridge (2009) found that 52% of dealers offered single nutrient controller-driven variable rate application of fertilizer. Yet only 11% of acres planted to winter wheat (Triticum aestivum L) used variable rate application technology for any fertilizing application (ARMS, 2012). Lack of adoption by producers may be due to the marginal economic feasibility of PS.

With respect to nitrogen (N) fertilizer application, economic feasibility of PS hinges on the capability of the technology to either 1) increase yields by recommending more N, or 2) retaining yields while recommending less N. Increasing yields by applying more N is an unlikely strategy for PS technology, as producers already apply more N than necessary i[1]n most years (El-Hout and Blackmer, 1990; Babcock, 1992). Thus, economic feasibility, and the main reason for PS technology adoption, hinges on PS technology retaining yields while recommending less N than would be applied without the technology.

The Raun et al. (2002, 2005) precision sensing system uses plant sensing of hard red winter wheat in February to determine recommended levels of topdress N. Previous research indicated that the Raun et al. PS system did lower input costs by recommending less N than farmer practice; however, the PS system recommended level of N did not retain yield levels (e.g., Biermacher et al., 2009; Boyer et al., 2011). The yield loss from the PS system recommended N could be due to an implicit assumption of zero prediction error, which is unlikely since weather after sensing affects potential yields

[1] Note that this is an expected profit maximizing strategy as the nitrogen will be needed in some years. The marginal revenue of applying nitrogen when it is needed is roughly 6-10 times its marginal cost and thus it pays to apply nitrogen that is only needed once every 6-10 years.

Information that is currently not directly used by a PS system is information a producer has about a given field[2]. A producer does not possess perfect information about the functional relationship of expected yield and N. Instead, a producer has some knowledge of past N and yield levels and it is likely the error associated with an N choice decreases as a priori information increases. A Bayesian approach allows a producer to combine optical sensing information with prior objective – or subjective – information to decrease the error associated with the calculation of profit-maximizing N.

A Bayesian approach has been applied in determining the economic value of weather information to agricultural producers (e.g., Doll, 1971; Baquet, Halter, and Conklin, 1976; Byerlee and Anderson, 1982; Marshall, Parton, and Hammer, 1996), projecting agricultural yield expectations (e.g., Krause, 2008), and determining returns of using soil sample information (e.g., Pautsch, Babcock, and Breidt, 1999). However, there has been no research on how Bayesian updating can improve profit-maximizing N recommendations by a PS system.

We develop Bayesian methods to combine prior information with PS information. The results should be beneficial to wheat producers as N can have the largest effect on yield (Johnston, 2000) -- among chemicals -- and fertilizer costs may account for more than 30 percent of operating costs (Huang, McBride, and Vasavada 2009). Furthermore, advancement in PS profit-maximizing N recommendations may increase the adoption rate of precision agriculture, and thus benefit manufacturers of precision agriculture equipment. Moreover, increased adoptions of PS will likely decrease N application levels and consequently reduce negative environmental externalities associated with excess N application.

2. Theoretical Framework for Applying a Bayesian Approach to Parameters of a Production Function

2.1. Prior Information

Consider a risk neutral producer who maximizes expected profit with a choice set that is reduced to an N decision. The objective function for a producer can be represented by

(1) $$\max_{N_{it}} E(\pi_{it}|N_{it}, I_0) = \max_{N_{it}} pE(Y_{it}) - rN_{it}$$

$$s.t. E(Y_{it}) = \int Y_{it}(N_{it};\Theta)\, f(\Theta|I_0)d\Theta$$

where $E(\pi_{it}|N_{it}, \Phi)$ is expected profit, p and r are known output price and N cost, and $E(Y_{it})$ is expected yield as a function of N, represented by N_{it}, prior information set I_0, and functional relationship of expected yield and N with parameter vector Θ.

Previous research has generally favored plateau-type models over polynomial functions. We use the linear stochastic plateau of Tembo et al. (2008) since it was developed to match the production function assumed with Raun et al.'s (2005) precision nitrogen system. Tembo et al.'s stochastic plateau model has been used in many previous studies to estimate yield response to N (Biermacher et al., 2009; Tumusiime et al., 2011; Boyer et al. 2013) and can be expressed as

(2) $$Y_{it} = \min(\beta_{0it} + \beta_{1it}N_{it}, \mu_{Pit} + v_{it}) + \tau_{it} + \varepsilon_{it},$$

where Y_{it} is the observed yield of location i in year t, N_{it} is level of N applied, $v_{it} \sim N(0, \sigma_v^2)$ is the plateau year random effect, $\tau_{it} \sim N(0, \sigma_\tau^2)$ is the year random effect, $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$ is a random error term, and β_{0it}, β_{1it}, and μ_{Pit} (mean plateau yield) are parameters to be estimated. The stochastic variables (v_{it}, τ_{it}, ε_{it}) are assumed to be independent. A yield function estimated with data from previous years does not consider information about current growing conditions.

[2] Producers will sometimes intuitively apply a little more than the recommendation if it is less than their current practice or a little less if the recommendation is more, but they are not using an optimal Bayesian approach.

A producer likely thinks either in terms of expected yield, or a level of nitrogen, rather than in terms of the parameters of a production function. In practice, the parameters would need to be calibrated based on the producers expected yield and expected optimal level of nitrogen, as well as historical information for the field or region. For our research, historical data are available and thus an objective prior can be used.

2.2. New information

There are many PS systems available that vary by sensor type and nitrogen response index (Alchanatis, Scmilovitch, and Meron, 2005; Begiebing et al., 2007; Ehlert, Schmerler, and Voelker, 2004; Havránková et al., 2007). The N fertilizer optimization algorithm (NFOA) developed by Raun et al. (2002) and updated by Raun et al. (2005) has been produced commercially under the name GreenSeeker®, which uses an optical sensor and recommends a level of N based on mid-season growth. The PS algorithm used here corresponds closely to that of GreenSeeker®.

The Raun et al. (2002, 2005) NFOA procedure can be separated into three processes: 1) estimate potential yield if no additional mid-season N is applied, 2) estimate maximum potential yield if additional mid-season N is applied, 3) calculate the required additional mid-season N to reach maximum potential yield. Note that the first and second processes of NFOA are essentially that same information as β_0 and μ_P in equation (2), but NFOA does not provide information similar to β_1. Also, note that producers could also easily be asked to provide subjective estimates of their yield goal and the amount of nitrogen to reach that yield goal, which could be used to derive the same parameters in equation (2).

Processes one and two of NFOA are achieved by applying a non-yield-limiting amount of N to a narrow strip in the field before planting – also referred to as an N-rich strip. Then between Feekes growth stages 4 and 6 optical reflective measurements (ORM) are used to calculate a normalized difference vegetation index (NDVI) that measures the total biomass produced by the plant at the time of measurement. NDVI is divided by the number of growing days to arrive at an in-season estimated grain yield (INSEY). INSEY where the field-level pre-plant N has been applied is used to determine minimum potential yield and INSEY where the non-yield-limiting pre-plant N has been applied is used to determine maximum potential yield of a given field.

Raun et al. (2002, 2005) use an exponential functional form for the relationship between yield and INSEY. We use a linear functional form since there is little difference in the explanatory power of the two functional forms and the linear functional form leads to a simpler computation for the Bayesian optimization problem. To calculate minimum and maximum yield potential for a given location and year, the linear model used was

$$Y_{it} = \gamma_{0it} + \gamma_{1it} INSEY_{it} + z_{it} + \epsilon_{it}, \tag{3}$$

where $INSEY_{it}$ is INSEY for location i in year t, $z_{it} \sim N(0, \sigma_z^2)$ is the year random effect, $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$ is a random error term, and γ_{0it} and γ_{1it} are parameters to be estimated. Estimated minimum yield potential, represented by $\widehat{YP}_{Oit}$, and maximum yield potential, represented by $\widehat{YP}_{Mit}$, for location i in year t is found by

$$\widehat{YP}_{Oit} = \hat{\gamma}_{0it} + \hat{\gamma}_{1it} \overline{INSEY}_{Oit} \tag{4}$$

and

$$\widehat{YP}_{Mit} = \hat{\gamma}_{0it} + \hat{\gamma}_{1it} \overline{INSEY}_{Mit} \tag{5}$$

where $\overline{INSEY}_{Oit}$ is average INSEY where felid-level pre-plant N was applied for location i in year t, $\overline{INSEY}_{Mit}$ is average INSEY across plots where non-yield-limiting pre-plant N was applied for location i in year t, and $\hat{\gamma}_{0it}$ and $\hat{\gamma}_{1it}$ are the estimated parameters from equation (5).

2.3. Data used for Estimating Parameters

Data from hard red winter wheat experiments conducted on agronomic research stations in Lahoma and Stillwater, OK, USA consisting of yields, amounts of N, and INSEY from ORM were used to estimate parameters. Observations on N applied and yield were collected in Lahoma from 1971 to 2012 and Stillwater OK from 1969 to 2012. Observations on PS variables were also collected at these locations; however, PS technology is a relatively recent invention and thus PS data collection did not begin until 1999. The Stillwater location is missing observations for 2007, 2009, and 2010.

Parameters for the prior information (i.e., equation (2)) and new information (i.e., equation (3)) were estimated for every year that both sensing and yield information were available using an out-of-sample approach, similar to Norwood, Lusk, and Brorsen (2004). That is, data collected from a given year were excluded when estimating parameters for that year. On the contrary, the average INSEY values (i.e., $\overline{INSEY}_{Oit}$ and $\overline{INSEY}_{Mit}$) were calculated for a given year using data only from that year.

2.4. Combining Prior and New Information with Bayesian Updating

As shown in Figure 1, the prior and new information contain analogous information, with the exception that new information does not provide an estimate of yield response to N. The process of using new information to revise prior information is the essence of Bayesian updating. Figure 2, adapted from Zellner (1971), schematically represents how prior information is used to estimate a prior density, new information is used to estimate a likelihood function, and the prior density and likelihood function are combined using Bayes' Theorem to get a posterior density.

The benefit of relatively quick computation is essential for a PS system that has limited computing power and may be used on the go. Therefore, we assume all densities are distributed multivariate normal.

Prior information (i.e., historical yield and N levels), represented by I_0, can be used to estimate the relationship between expected yield and N in equation (2) and the vector of estimated parameters for the prior information is represented by

$$\boldsymbol{\varphi}_0 = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\mu}_t \end{pmatrix}, \tag{6}$$

with covariance matrix

$$\boldsymbol{\Sigma}_0 = \begin{pmatrix} \mathrm{var}(\hat{\beta}_0) & \mathrm{cov}(\hat{\beta}_0, \hat{\beta}_1) & \mathrm{cov}(\hat{\beta}_0, \hat{\mu}_t) \\ \mathrm{cov}(\hat{\beta}_1, \hat{\beta}_0) & \mathrm{var}(\hat{\beta}_1) & \mathrm{cov}(\hat{\beta}_1, \hat{\mu}_t) \\ \mathrm{cov}(\hat{\mu}_t, \hat{\beta}_0) & \mathrm{cov}(\hat{\mu}_t, \hat{\beta}_1) & \mathrm{var}(\hat{\mu}_t) + \hat{\sigma}_v^2 \end{pmatrix}. \tag{7}$$

The prior density then is represented by $p(\boldsymbol{\Theta}_0)$ and is distributed $N(\boldsymbol{\varphi}_0, \boldsymbol{\Sigma}_0)$.

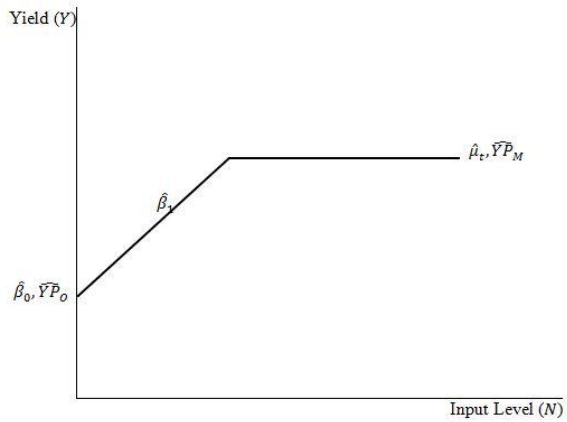

Figure 1. Representation of the relationship for parameters of the prior information and new information.

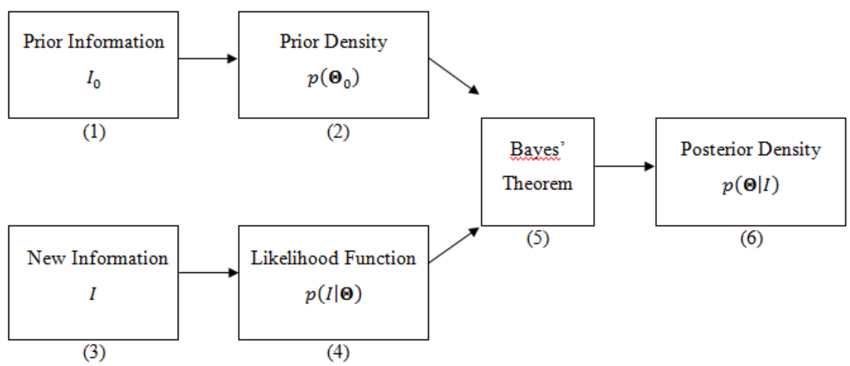

Figure 2. The process of using new information to revise prior information.

New information (i.e., PS), represented by I, can be used to estimate the minimum and maximum yield potential in equations (6) and (7). However, I does not measure yield response to additional N and therefore the vector of estimated parameters for the new information must be supplemented with the slope estimate. The new data is noninformative with respect to the slope parameter. Thus, the vector of estimated parameters for the new information is

(8)
$$\boldsymbol{\varphi} = \begin{pmatrix} \widehat{YP}_O \\ \hat{\beta}_1 \\ \widehat{YP}_M \end{pmatrix},$$

with covariance matrix $\boldsymbol{\Sigma}$ equal to

$$\begin{pmatrix} var(\hat{\gamma}_0) + var(\hat{\gamma}_1)\overline{INSEY_O}^2 + 2cov(\hat{\gamma}_0, \hat{\gamma}_1)\overline{INSEY_O} + \sigma_\varepsilon^2 + \sigma_\varepsilon^2 & 0 & var(\hat{\gamma}_0) + [\overline{INSEY_O} + \overline{INSEY_M}]cov(\hat{\gamma}_0, \hat{\gamma}_1) + [\overline{INSEY_O} * \overline{INSEY_M}]var(\hat{\gamma}_1) + \sigma_\varepsilon^2 \\ 0 & \infty & 0 \\ var(\hat{\gamma}_0) + [\overline{INSEY_O} + \overline{INSEY_M}]cov(\hat{\gamma}_1, \hat{\gamma}_0) + [\overline{INSEY_O} * \overline{INSEY_M}]var(\hat{\gamma}_1) + \sigma_\varepsilon^2 & 0 & var(\hat{\gamma}_0) + var(\hat{\gamma}_1)\overline{INSEY_M}^2 + 2cov(\hat{\gamma}_0, \hat{\gamma}_1)\overline{INSEY_M} + \sigma_\varepsilon^2 + \sigma_\varepsilon^2 \end{pmatrix}$$

The likelihood function then is represented by $p(I|\boldsymbol{\Theta})$ and distributed $N(\boldsymbol{\varphi}, \boldsymbol{\Sigma})$. In practice, infinity is approximated with a large number to make the calculations tractable.

The posterior vector of estimated parameters is a linear combination of the prior and likelihood vectors of estimated parameters (i.e., $\boldsymbol{\varphi}_0$ and $\boldsymbol{\varphi}$). More specifically, the prior vector of estimated parameters is multiplied by the covariance matrix of the likelihood (i.e., $\boldsymbol{\Sigma}$) and added to the product of the likelihood vector of estimated parameters and the prior covariance matrix (i.e., $\boldsymbol{\Sigma}_0$). Both terms of the linear combination are normalized by the sum of the covariance matrices (i.e., $\boldsymbol{\Sigma}_0$ and $\boldsymbol{\Sigma}$). Thus, the posterior mean vector is a weighted average of the prior and likelihood mean vectors and the information with lower variance has a greater weight. Mathematically defined, the posterior vector of estimated parameters - derived formally in Duda, Hart, and Stork (2001, pp. 95-97) - is

(9) $$\boldsymbol{\varphi}_b = \boldsymbol{\Sigma}_0 \left(\boldsymbol{\Sigma}_0 + \tfrac{1}{n}\boldsymbol{\Sigma}\right)^{-1} \boldsymbol{\varphi} + \tfrac{1}{n}\boldsymbol{\Sigma}\left(\boldsymbol{\Sigma}_0 + \tfrac{1}{n}\boldsymbol{\Sigma}\right)^{-1} \boldsymbol{\varphi}_0$$

with covariance matrix

(10) $$\boldsymbol{\Sigma}_b = \boldsymbol{\Sigma}_0 \left(\boldsymbol{\Sigma}_0 + \tfrac{1}{n}\boldsymbol{\Sigma}\right)^{-1} \tfrac{1}{n}\boldsymbol{\Sigma}$$

Note that (10) and (11) assume that $\boldsymbol{\Sigma}_0$ and $\boldsymbol{\Sigma}$ are known whereas we are using estimates. Then, using $\boldsymbol{\mu}_b$ and the Cholesky decomposition $\boldsymbol{\Sigma}_b$, the posterior density is simulated using Monte Carlo simulation with 10,000 iterations. The posterior density then is represented by $p(\boldsymbol{\Theta}|I)$ and distributed $N(\boldsymbol{\varphi}_b, \boldsymbol{\Sigma}_b)$.

2.5. An Example of Estimated and Combined Parameters

For illustration, Table 1 shows parameter estimates for the prior density and likelihood function and the calculation of the posterior density for Stillwater in 2012. The minimum and maximum yield potential in the posterior mean vector are bounded by the minimum and maximum yield potential in the prior and likelihood mean vectors. Thus, the posterior density mean vector is a weighted average of the two sources of information.

A benefit of Bayesian updating that should not be overlooked can be seen by comparing the covariance matrices. Combining the prior and new information decreases the variance associated with estimating minimum and maximum yield potential.

Table 1. Parameter Estimates for Stillwater in 2012

Parameter	Estimate	Standard Error
β_0	18.09***[a]	1.16
β_1	0.22***	0.03
μ_p	27.07***	1.57
σ_v^2	109.64***	26.25
σ_t^2	27.14***	1.64
γ_0	1.70	3.03
γ_1	3,563.64***	373.94
σ_z^2	34.17*	16.52
$\overline{INSEY}_{Oit}$	0.01	0.00[b]
$\overline{INSEY}_{Mit}$	0.01	0.00[b]
$\widetilde{YP}_{Oit}$	19.84	9.86
$\widetilde{YP}_{Mit}$	25.14	9.85

$$\boldsymbol{\varphi}_0 = \begin{pmatrix} 18.09 \\ 0.22 \\ 27.07 \end{pmatrix} \quad \boldsymbol{\varphi} = \begin{pmatrix} 19.84 \\ 0.22 \\ 25.14 \end{pmatrix} \quad \boldsymbol{\varphi}_b = \begin{pmatrix} 18.51 \\ 0.22 \\ 25.96 \end{pmatrix}$$

$$\boldsymbol{\Sigma}_0 = \begin{pmatrix} 110.99 & -0.02 & 110.84 \\ -0.02 & 0.00 & -0.02 \\ 110.84 & -0.02 & 204.87 \end{pmatrix} \quad \boldsymbol{\Sigma} = \begin{pmatrix} 97.16 & 0 & 37.82 \\ 0 & 10E6 & 0 \\ 37.82 & 0 & 96.98 \end{pmatrix} \quad \boldsymbol{\Sigma}_b = \begin{pmatrix} 46.71 & -0.01 & 29.74 \\ -0.01 & 0.00 & -0.01 \\ 31.46 & -0.01 & 65.09 \end{pmatrix}$$

a: ***, **, and * represent 0.01, 0.05, and 0.10 levels of statistical significance, respectively.

b: Standard deviation, not a standard error.

3. N Recommendations

N recommendation following Tembo et al. (2008) were denoted by N_i^{PI}, and a Bayesian N recommendation, denoted by N_i^{B}, using prices from the United States Department of Agriculture National Agricultural Statistics Service (NASS, 2013a; NASS, 2013b). The price of a pound of urea for the 2013 marketing year was \$0.64 and this was used for the cost of N (r) and the price received for a bushel of wheat (p) in 2013 marketing year was \$6.89.

However, equating value of marginal expected product to the price of N may cause a producer to apply too much N. For example if $r/(p\beta_1) = 0.2$, then in four out of five years producers would be applying more nitrogen than was needed in that year. The optimal expected yield will be less than the expected plateau yield since in this example, in one year out of five, the maximum yield would not be reached. A perfect-information PS system would be able to achieve the plateau yield with an expected level of nitrogen of $(\mu_P - \beta_0)/\beta_1$, but any imperfect information system will have a lower yield and use more nitrogen. Nevertheless, the N recommendation for new information was found by

(11) $$N_i^{NI} = \frac{\widehat{YP}_M - \widehat{YP}_O}{\hat{\beta}_1}.$$

New information recommendations were restricted to a nonnegative number as yield was greater when no N was applied in some years. The approach in (15) is essentially what was proposed by Raun et al. (2002; 2005). The approach led to too little nitrogen being applied, so with experience $\hat{\beta}_1$, which is nitrogen use efficiency, has been reduced so that current formulas have been heuristically adjusted for the uncertainty. This heuristic adjustment is not considered here.

4. Yield and Profitability Comparisons

A linear plateau was estimated as a yield function with respect to N for both locations and for each year that both prior and new information were available. The linear plateau is

(12) $$y_i = \min(\delta_0 + \delta_1 N_i, \tau) + \omega_i,$$

where y_i is the observed yield of plot i, N_i is level of N applied to plot i, $\omega_i \sim N(0, \sigma_\omega^2)$ is a random error term, and δ_0, δ_1, and τ (plateau yield) are parameters to be estimated. The N recommendations (i.e., N_i^{PI}, N_i^{NI}, and N_i^{B}) were then plugged into the estimated linear plateau to arrive at the estimated yields. Additional to the estimated yields for the N recommendations, yields were also estimated for a constant application of 60 and 90 pounds of N.

To determine economic feasibility, net returns were calculated for each estimated yield and corresponding N recommendation. Custom urea application rate of \$4.85 estimated by Doye and Sahs (2014) was used for application cost.

N recommendations and net returns for Lahoma and Stillwater are shown in Tables 2 and 3, respectively. On average, in pounds per acre, the prior information, new information, and Bayesian N recommendations were 59.33, 33.92, and 54.51 in Lahoma and 50.48, 28.14, and 42.13 in Stillwater. The average Bayesian N recommendation was not significantly different than the average prior information N recommendation in Lahoma (F-test, p-value=0.14); however, a Bayesian approach recommended less N than prior information in Stillwater (F-test, p-value=0.03). On average, for both locations, new information N recommendations were significantly lower than both prior information and Bayesian N recommendations (F-test, p-value<0.01, for both locations).[3] This confirms that the new information, NFOA, tends to recommend lower levels of N. The new information N recommendation had more variation than prior information and Bayesian N recommendations. However, this is partly due to new information to recommending zero N some years. A recommendation of zero N has the benefit of saving application costs (i.e., $4.85).

Average net returns in dollars per acre for prior information, new information, and Bayesian N recommendations were 279.36, 270.02, and 279.33 in Lahoma and 143.33, 139.43, and 143.06 in Stillwater. Additionally, the net returns for a constant application of 60 and 90 pounds of N were 279.10 and 289.25 in Lahoma and 149.17 and 149.47 in Stillwater. The only significant difference in net returns in Lahoma was between the new information N recommendation and 90lb of N (F-test, p-value=0.03). There were no significant differences in net returns across N recommendation in Stillwater.

Table 2. Lahoma N Recommendations and Net Returns

	N Recommendation (lb/ac)			Net Returns ($/ac)				
Year	Prior Information	New Information	Bayesian	Prior Information	New Information	Bayesian	60lb of N	90lb of N
1999	55.42	25.80	49.52	282.00	211.65	267.98	292.88	364.12
2000	57.63	50.35	64.94	318.29	306.64	313.59	316.77	297.48
2001	63.86	0	48.45	124.20	151.66	134.10	126.68	107.39
2002	57.57	5.45	44.44	273.26	288.19	281.71	271.70	252.41
2003	57.24	47.34	58.94	465.59	437.85	470.34	473.32	537.42
2004	57.77	12.54	45.31	215.69	221.43	223.70	214.26	194.97
2005	57.70	31.74	52.63	232.96	221.67	236.22	231.48	212.19
2006	57.90	7.82	40.20	214.64	220.40	216.68	214.40	210.95
2007	57.54	13.76	43.81	273.30	278.54	274.95	273.01	269.42
2008	57.46	56.96	62.15	520.65	520.97	517.62	519.01	499.72
2009	62.91	48.51	56.26	300.91	262.27	283.08	293.11	373.60
2010	62.74	78.72	75.36	124.54	135.78	133.41	122.61	143.70
2011	62.90	46.27	61.79	215.46	207.30	214.92	214.04	228.77
2012	61.97	49.67	59.33	349.60	315.88	342.36	344.19	357.35
Average	59.33	33.92	54.51	279.36	270.02	279.33	279.10	289.25
Standard Deviation	2.73	22.64	9.53	107.71	100.56	104.88	108.24	120.69

[3] Note that current implementation of NFOA recognizes that the plug-in approach leads to under application of N. To correct the problem, current models use a lower value of β_1 in order to get closer to the optimal level.

Table 3. Stillwater N Recommendations and Net Returns

	N Recommendation (lb/ac)			Net Returns ($/ac)				
Year	Prior Information	New Information	Bayesian	Prior Information	New Information	Bayesian	60lb of N	90lb of N
1999	44.75	0	25.98	110.11	87.71	98.68	119.39	112.71
2000	48.67	67.05	59.67	174.09	198.31	188.59	189.03	228.56
2001	43.77	7.18	33.62	134.66	109.01	130.57	124.22	104.93
2002	48.94	39.74	44.54	152.28	141.26	147.01	165.52	201.45
2003	49.77	40.18	46.18	171.00	161.64	167.50	180.99	210.29
2004	59.71	0	27.79	141.19	204.20	172.29	140.91	111.69
2005	50.90	35.44	48.69	181.77	181.96	181.80	181.67	170.19
2006	59.17	33.24	50.14	33.83	54.81	41.66	33.12	7.10
2008	47.88	9.14	33.73	193.79	147.56	176.90	208.25	213.65
2011	52.52	53.57	54.58	81.77	81.95	82.12	83.04	88.13
2012	49.16	24.02	38.55	202.10	165.28	186.55	214.71	195.42
Average	50.48	28.14	42.13	143.33	139.43	143.06	149.17	149.47
Standard Deviation	4.84	21.15	10.51	48.83	47.45	46.90	53.20	66.13

Table 4 shows how changes in the price received for a bushel of wheat effects net returns for the various N recommendations. Changes in price received does effect the ordering of net returns. For example, at the low wheat price (i.e., $3.45/bushel), an N recommendation from new information has the highest net return for both Lahoma and Stillwater. However, the only additional significant difference in net returns found from the sensitivity analysis was between the new information N recommendation and 90lb of N at the high price (i.e., $10.34/bushel) for Stillwater (*F*-test, *p*-value=0.04).

Table 4. Sensitivity Analysis of Net Returns for a Change in Wheat Price

	Lahoma Net Returns ($/ac)					Stillwater Net Returns ($/ac)				
Price ($/bu)	Prior	New	Bayes	60lb of N	90lb of N	Prior	New	Bayes	60lb of N	90lb of N
10.34	449.4	418.2	445.4	440.4	465.2	246.5	220.2	243.2	245.5	255.6
6.89	279.4	270.0	279.3	279.1	289.2	143.3	139.4	143.1	149.2	149.5
3.45	116.9	121.8	119.6	117.8	113.3	57.8	58.7	58.0	52.9	43.4
Average	281.9	270.0	281.5	279.1	289.3	149.2	139.4	148.1	149.2	149.5
SD	135.7	121.0	133.0	131.7	143.7	77.2	65.9	75.7	78.6	86.6

5. Discussion

As the use of technology in agriculture increases, so will the economic value of technological knowledge. This does not render the knowledge of production practices by a producer useless. However, this is exactly the implications of any PS system that does not incorporate a producer's prior information when formulating an N recommendation. Alternatively, new technology is capable of increasing the information set for a producer and will likely continue to provide more accurate N recommendations.

This paper establishes a method Bayesian to combine PS and prior information about the response to N for a given field. Results suggest that PS technology recommended a lower level of N than standard practice or using Bayesian updating. Thus, increased adoptions of PS may have social benefits by reducing negative environmental externalities associated with excess N application, such as runoff into waterways and increased carbon emissions, without sacrificing net returns.[4]

The approach taken in this paper is noisy, as PS data was not available prior to 1999 and the yield function used to evaluation the N recommendations was from one year of data. Moreover, data for the prior information may include an unobserved technological change in yield response to N. Nevertheless, the method within provides a foundation for Bayesian updating parameters of a yield function. Future research could build on this study by examining the differences between Bayesian updating of parameters to obtain an N recommendation, as done here, and Bayesian updating of N recommendations derived from separate sources of information.

6. Acknowledgements

Brorsen receives financial support from the Oklahoma Agricultural Experiment Station and USDA National Institute of Food and Agriculture, Hatch Project number OKL02939 and the A.J. and Susan Jacques Chair. The authors also wish to thank W.R. Raun for providing the data for this research.

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[4] It is important to note that the investment of PS technology has not been taken into account in calculating net returns.

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